rel max

f not cont

Increasing/Decreasing Test:

If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a, b)

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on (a, b)

First derivative test:

Suppose c is a critical number of a continuous function f, then

f' $-$ c $+$ ~~local~~ rel min $+$ c $-$ $+$ c $+$ $-$ c $-$

$\boxed{\text{rel min}}$ $\boxed{\text{rel max}}$ no rel max/min

Defn: f is **concave down** if the graph of f lies below the tangent lines to f. **f' is decreasing**

Defn: f is **concave up** if the graph of f lies above the tangent lines to f. **f' is increasing**

Concavity Test:

If $f''(x) > 0$ for all $x \in (a, b)$, then f is concave upward on (a, b).

If $f''(x) < 0$ for all $x \in (a, b)$, then f is concave down on (a, b).

Defn: The point (x_0, y_0) is an **inflection point** if f is continuous at x_0 and if the concavity changes at x_0

inflection pt

inflection pt. Find using f''

Second derivative test: If f'' continuous at c, then

If $f'(c) = 0$ and $f''(c) > 0$, then f has a ~~local~~ relative minimum at c.

rel min

If $f'(c) = 0$ and $f''(c) < 0$, then f has a ~~local~~ relative maximum at c.

rel max

If $f'(c) = 0$ and $f''(c) = 0$, second derivative test gives no info.

If f is increasing on (a,b), then $f'(x) > 0$ on (a,b) T **F**

FALSE STATEMENT

Converses are not true:

Increasing/Decreasing Test

If $f'(x) > 0$ for all $x \in (a,b)$, then f is increasing on (a,b)

f increasing on (a,b) <u>does not imply</u> $f'(x) > 0$ for all $x \in (a,b)$.

Ex: $f(x) = x^3$ is an increasing fn

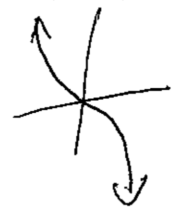
$f'(x) = 3x^2$, $f'(0) = 0$

If $f'(x) < 0$ for all $x \in (a,b)$, then f is decreasing on (a,b)

f decreasing on (a,b) <u>does not imply</u> $f'(x) < 0$ for all $x \in (a,b)$.

Ex: $f(x) = -x^3$ is decreasing fn

$f'(x) = -3x^2$, $f(0) = 0$



Concavity Test:

If $f''(x) > 0$ for all $x \in (a,b)$, then f is concave upward on (a,b).

f concave upward on (a,b) <u>does not imply</u> $f''(x) > 0$ for all $x \in (a,b)$.

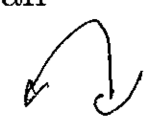
Ex: $f(x) = x^4$ is concave up

$f'(x) = 4x^3$, $f''(x) = 12x^2$, $f''(0) = 0$

If $f''(x) < 0$ for all $x \in (a,b)$, then f is concave down on (a,b).

f concave downward on (a,b) <u>does not imply</u> $f''(x) < 0$ for all $x \in (a,b)$.

Ex: $f(x) = -x^4$ is concave down



$f'(x) = -4x^3$, $f''(x) = -12x^2$

$f''(0) = 0$

Find the following for $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$ (if they exist; if they don't exist, state so). Use this information to graph f.

f' [1.5] 1a.) critical numbers: $0, 2$

f'' f' [1.5] 1b.) relative maximum(s) occur at $x =$ 2

f'' f' [1.5] 1c.) relative minimum(s) occur at $x =$ 0

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

f'' [1.5] 1f.) Inflection point(s) occur at $x =$ -1

f' [1.5] 1g.) f increasing on the intervals $(0, 2)$ $\quad 0 < X < 2$

f' [1.5] 1h.) f decreasing on the intervals $(-\infty, 0) \cup (2, \infty) = \boxed{\begin{cases} X < 0 \text{ or} \\ X > 2 \end{cases}}$

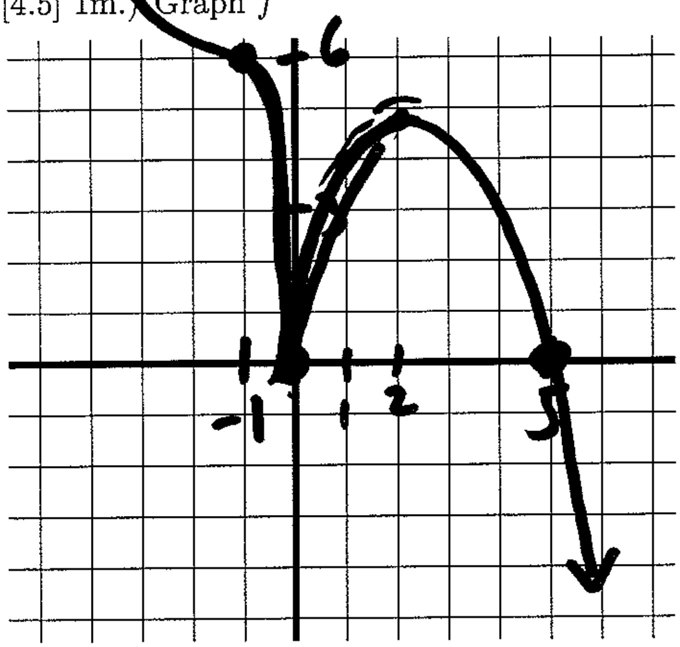
f'' [1.5] 1i.) f is concave up on the intervals $(-\infty, -1)$

f'' [1.5] 1j.) f is concave down on the intervals $(-1, 0) \cup (0, \infty)$ $\Leftarrow$ not concave down at $X = 0$

[1.5] 1k.) Equation(s) of vertical asymptote(s) none

[4] 1l.) Equation(s) of horizontal and/or slant asymptote(s) none

[4.5] 1m.) Graph f



$f(x) = 5x^{\frac{2}{3}} - x^{5/3}$

$f'(x) = \frac{10}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = 0, DNE$

$f''(x) = \frac{-10}{9}x^{-4/3} - \frac{10}{9}x^{-1/3} = 0, DNE$

---

f': $\frac{10}{3}x^{-1/3} - \frac{5}{3}x^{\frac{2}{3}} = 0, DNE$ **To find critical points**

$\frac{5}{3}x^{-1/3}(2 - x) = 0, DNE$

$\boxed{x = 0, 2}$ ← critical points

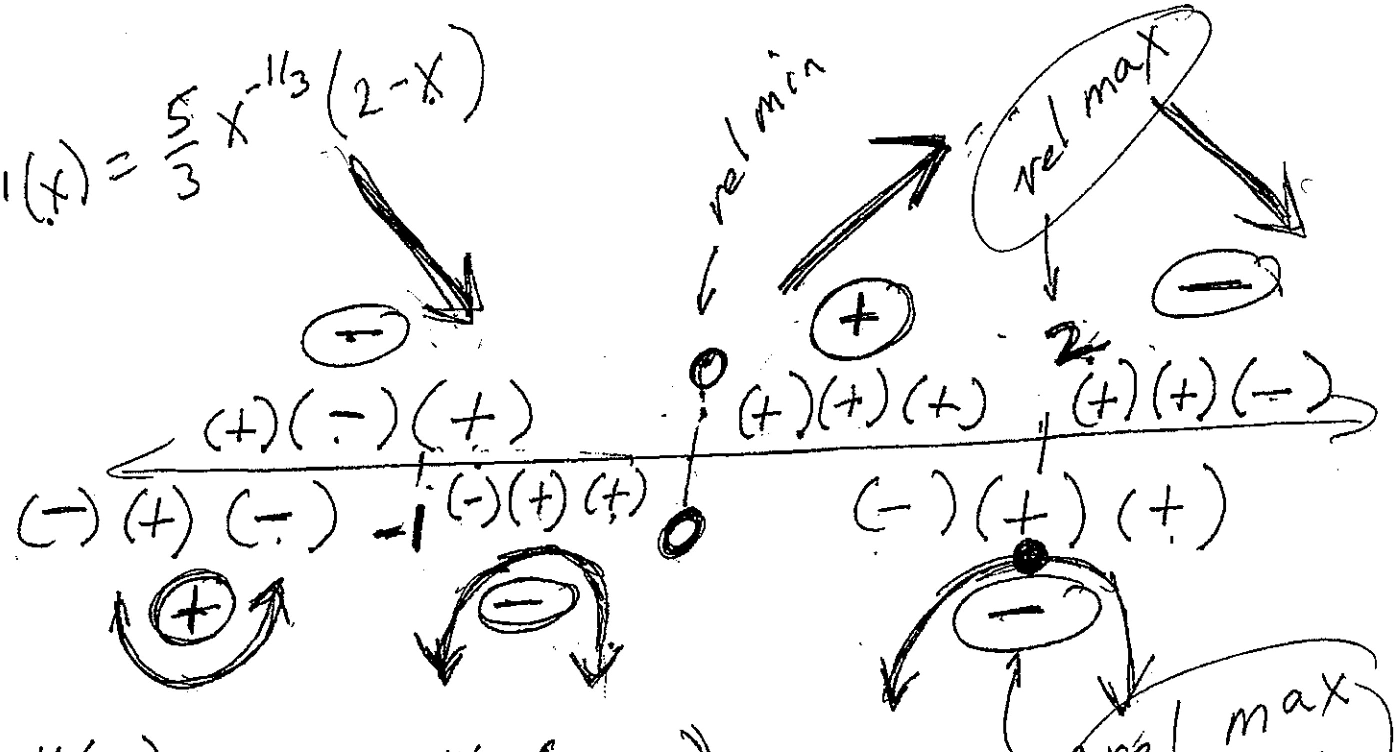
---

f'': $-\frac{10}{9}x^{-\frac{4}{3}} - \frac{10}{9}x^{-1/3} = 0, DNE$

$-\frac{10}{9}x^{-\frac{4}{3}}(1 + x) = 0, DNE$

$\boxed{x = 0, -1}$ ← from f''

$$f'(x) = \frac{5}{3}x^{-1/3}(2-x)$$



$$f''(x) = -\frac{10}{9}x^{-4/3}(1+x)$$

critical 0, 2

	x	$y = 5x^{\frac{2}{3}} - x^{5/3}$
critical	2	$5\sqrt[3]{4} - 2\sqrt[3]{4} = 3\sqrt[3]{4}$
	0	0
f''	−1	$5-(-1) = 6$
	5	0
	1	$5-1=4$

$$f(x) = 5x^{\frac{2}{3}} - x^{5/3}$$

$$= x^{2/3}(5-x) = 0$$

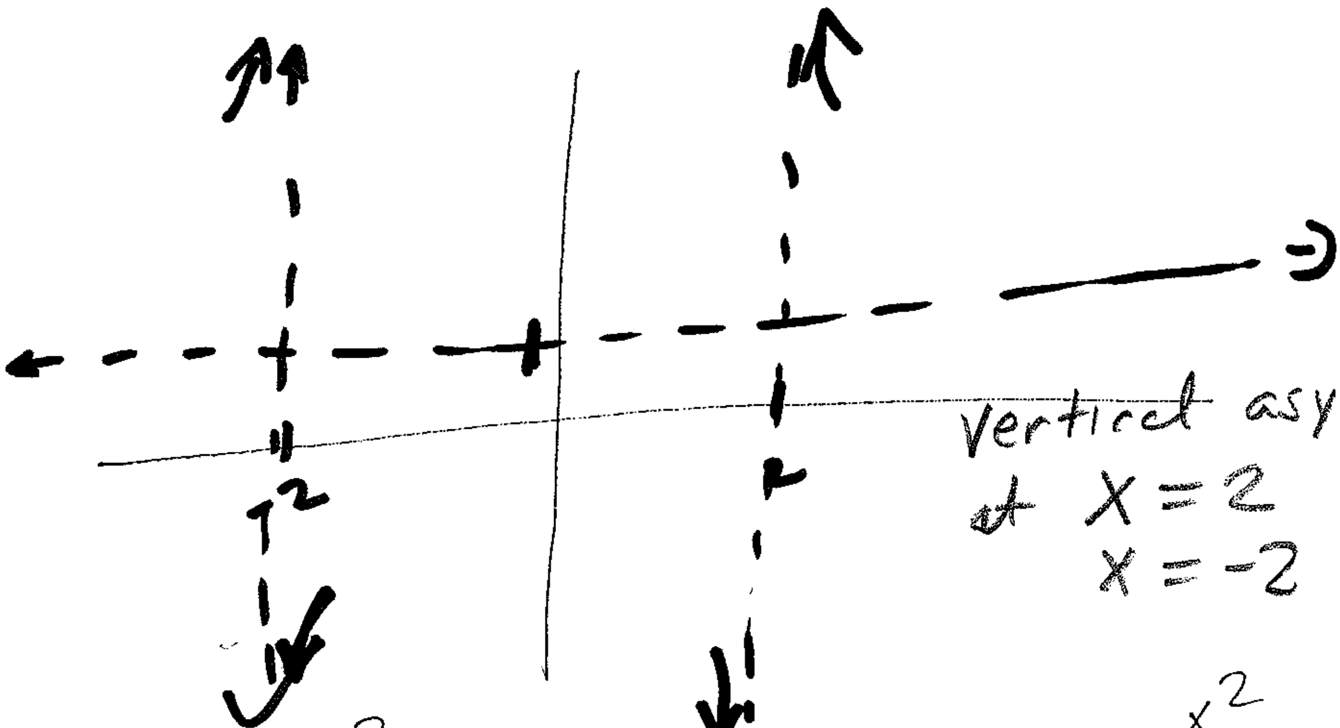
$$\frac{x^2}{x^2-4} \sim \frac{x^2}{x^2} = 1$$
for LARGE X

$y \to 1$ as $x \to +\infty$
$x \to -\infty$

Section 3.3:

horizontal asymptote at $y = 1$

Motivation: Graph $f(x) = \frac{x^2}{(x-2)(x+2)} = \frac{x^2}{x^2-4}$



vertical asymptote at $x = 2$
$x = -2$

$$\lim_{x \to 2^-} \frac{x^2}{(x-2)(x+2)} = -\infty$$

$$\text{"}\frac{4}{0^- \cdot 4}\text{"}$$

$$\lim_{x \to 2^+} \frac{x^2}{(x+2)(x-2)} = +\infty$$

$$\frac{\text{"}+\text{"}}{+\,0^+}$$

$$\lim_{x \to -2^-} \frac{x^2}{(x-2)(x+2)} = +\infty$$

$$\frac{\text{"}+\text{"}}{-\cdot\,0^-}$$

$$\lim_{x \to -2^+} \frac{x^2}{(x-2)(x+2)} = -\infty$$

$$\frac{\text{"}+\text{"}}{-\,0^+}$$

To find vertical asymptotes,

"+") find all $a \in \mathcal{R}$ such that

$lim_{x \to a^-} f(x) = \pm\infty$ and/or $lim_{x \to a^+} f(x) = \pm\infty$

Ex: $f(x) = \frac{1}{(x+2)(x-3)^2}$

$\lim_{x \to -2^-} \frac{1}{(x+2)(x-3)^2} = -\infty$

$\lim_{x \to -2^+} \frac{1}{(x+2)(x-3)^2} = +\infty$

vertical asymptotes $x = -2$, $x = 3$

$\lim_{x \to 3} \frac{1}{(x+2)(x-3)^2} = +\infty$

$\frac{\text{"}+\text{"}}{+\,0^+}$

note even exponent

Horizontal asymptotes/limits at infinity

To find horizontal asymptotes:

calculate $lim_{x \to +\infty} f(x)$ and $lim_{x \to -\infty} f(x)$

IF $lim_{x \to +\infty} f(x) = L$ where L is a finite real number, then $y = L$ is a horizontal asymptote.

IF $lim_{x \to -\infty} f(x) = K$ where K is a finite real number, then $y = K$ is a horizontal asymptote.

$\lim_{x \to +\infty} \frac{1}{(x+2)(x-3)^2} = 0$ $\boxed{y = 0}$

$\lim_{x \to -\infty} \frac{1}{(x+2)(x-3)^2} = 0$

dominate for large x for large X

Ex: $f(x) = \frac{2x^3 - x^2 + 1}{8x^3 + x + 3} \sim \frac{2\cancel{x^3}}{8\cancel{x^3}} = \frac{1}{4}$ plug in large values for X then $f(X)$ is approx by $\frac{1}{4}$

$lim_{x \to +\infty} \frac{2x^3 - x^2 + 1}{8x^3 + x + 3} = \frac{1}{4}$

Long optional method

$$\lim_{X \to +\infty} \frac{\cancel{X^3}(2 - \overset{0}{\frac{1}{X}} + \overset{0}{\frac{1}{X^3}})}{\cancel{X^3}(8 + \underset{0}{\frac{1}{X^2}} + \underset{0}{\frac{3}{X^3}})} = \frac{2}{8} = \frac{1}{4}$$

Similarly, $lim_{x \to -\infty} \frac{2x^3 - x^2 + 1}{8x^3 + x + 3} \sim \frac{2\cancel{x^3}}{8\cancel{x^3}} = \frac{1}{4}$

Horizontal asymptote(s):

$$\boxed{y = \frac{1}{4}}$$

dominate

Ex: $f(x) = \frac{x^2+1}{2x^5+x^2-3}$ $\sim \frac{x^2}{2x^5} = \frac{1}{2x^3}$

$lim_{x\to+\infty} \frac{x^2+1}{2x^5+x^2-3} = 0$

Similarly, $lim_{x\to-\infty} \frac{x^2+1}{2x^5+x^2-3} = 0$

Horizontal asymptote(s): $\boxed{y=0}$

← dominate

Ex: $f(x) = \frac{2x^5+x^2-3}{x^2+1}$

$lim_{x\to+\infty} \frac{2x^5+x^2-3}{x^2+1} = +\infty$

~~vert~~ $\frac{\text{very large}}{\text{not so large}} = \text{large}$

not finite

Also, $lim_{x\to-\infty} \frac{2x^5+x^2-3}{x^2+1} = -\infty$

Horizontal asymptote(s): NONE

Ex: $f(x) = \frac{2x}{\sqrt{x^2+1}} = \frac{2x}{|x|}$

$lim_{x \to +\infty} \frac{2x}{\sqrt{x^2+1}} = 2$

for large x

$\sqrt{x^2+1} \sim \sqrt{x^2} = |x|$

$lim_{x \to -\infty} \frac{2x}{\sqrt{x^2+1}} = -2$

Horizontal asymptote(s): $\boxed{y = 2, y = -2}$