3a) Find critical pts

$$C'(w) = -\frac{4000}{w^2} + 30 = 0, \text{DNE}$$

$$\frac{-4000 + 30w^2}{w^2} = 0, \text{DNE}$$

$w = 0$

$$-4000 + 30w^2 = 0$$

$$\frac{30w^2}{30} = \frac{400\cancel{0}}{3\cancel{0}}$$

$$w^2 = \frac{400}{3}$$

$$w = \sqrt{\frac{400}{3}}$$

(length > 0 so don't need $\pm$)

3b) Do we have abs min?

$$w \in (0, ?)$$
$$= (0, \infty)$$

Not closed interval so no EVT

$$C'(w) = -4000w^{-2} + 30$$

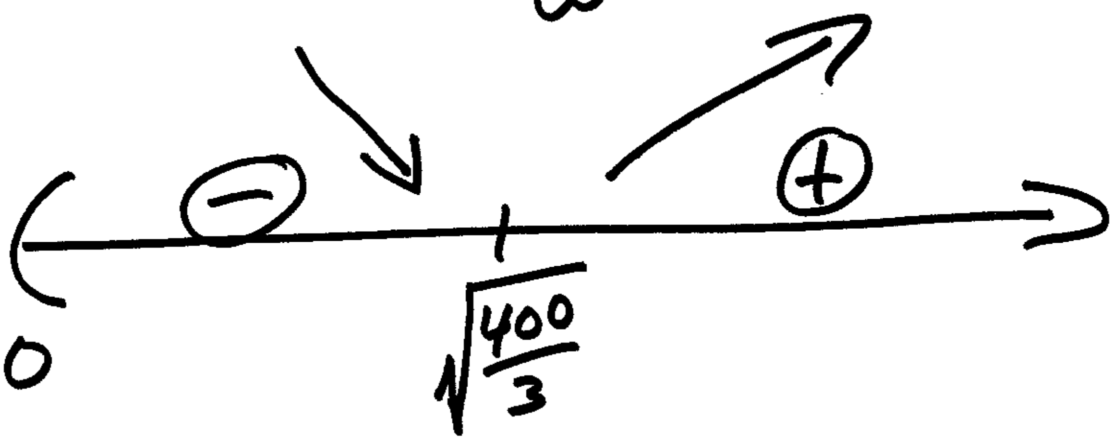
$$C''(w) = 8000w^{-3} > 0$$
for $w > 0$

We have only one critical pt in $(0, \infty)$

$C'\left(\sqrt{\frac{400}{3}}\right) = 0$. By thm 9, abs min at $w = \sqrt{\frac{400}{3}}$

First derivative

$$C'(w) = \frac{-4000}{w^2} + 30 = \frac{-4000+30w^2}{w^2}$$



$$\Rightarrow \text{abs min at } w = \sqrt{\frac{400}{3}}$$

$$l = 400 \cdot \sqrt{\frac{3}{400}} = \sqrt{400 \cdot 3}$$

$$= 20\sqrt{3} \ m$$

Dimension $w = \frac{20}{\sqrt{3}}$ m, $l = 20\sqrt{3}$ m

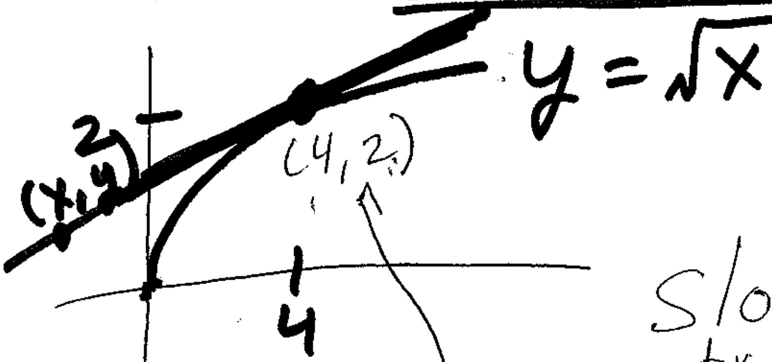
where w is side next to river

3.6

Find the linearization of $\sqrt{x}$ at $x = 4$

I.e, find the best linear approximation of $\sqrt{x}$ for x close to 4.

I.e, find the equation of tangent line to $\sqrt{x}$ at $x = 4$.



$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-1/2}$

Slope at $x=4$ $= f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f(4) = \sqrt{4} = 2$

pt of line

$\frac{y-2}{x-4} = \frac{1}{4} \Rightarrow y - 2 = \frac{1}{4}(x-4)$

$y - 2 = \frac{1}{4}x - 1$

$\boxed{y = \frac{1}{4}x + 1}$

Approximate $\sqrt{5}$

Method 1: Use equation of tangent line

Let $g(x) = \frac{1}{4}x + 1$ ⟸ tangent line

$\sqrt{5} = f(5) \sim g(5) = \frac{5}{4} + 1 = \frac{9}{4}$

sine $f(x) \sim g(x)$ for x near 4

$\sqrt{5} \sim \frac{9}{4}$

Method 2 (optional, but quicker): Use $\Delta y \sim dy$

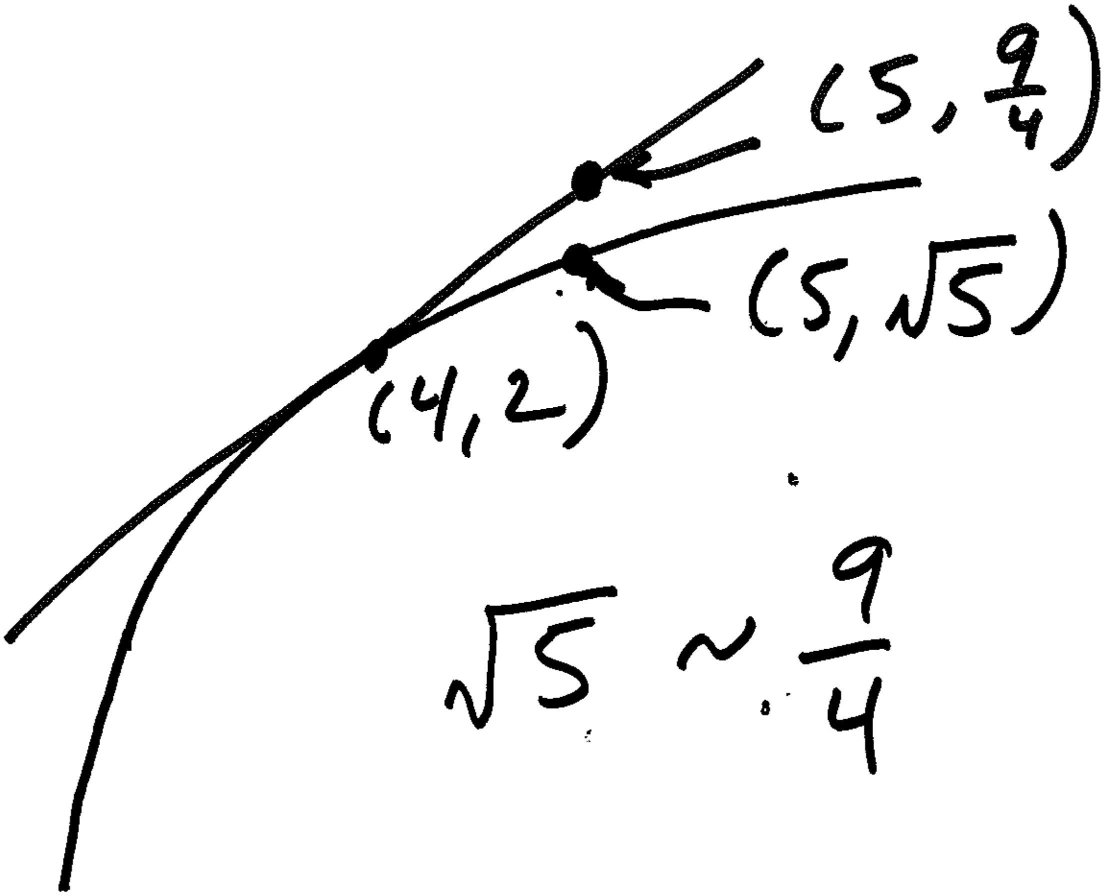
Recall: slope of secant line $= \frac{\Delta y}{\Delta x}$

$\Delta x = x + h - x, \quad \Delta y = f(x+h) - f(x) = f(x + \Delta x) - f(x)$

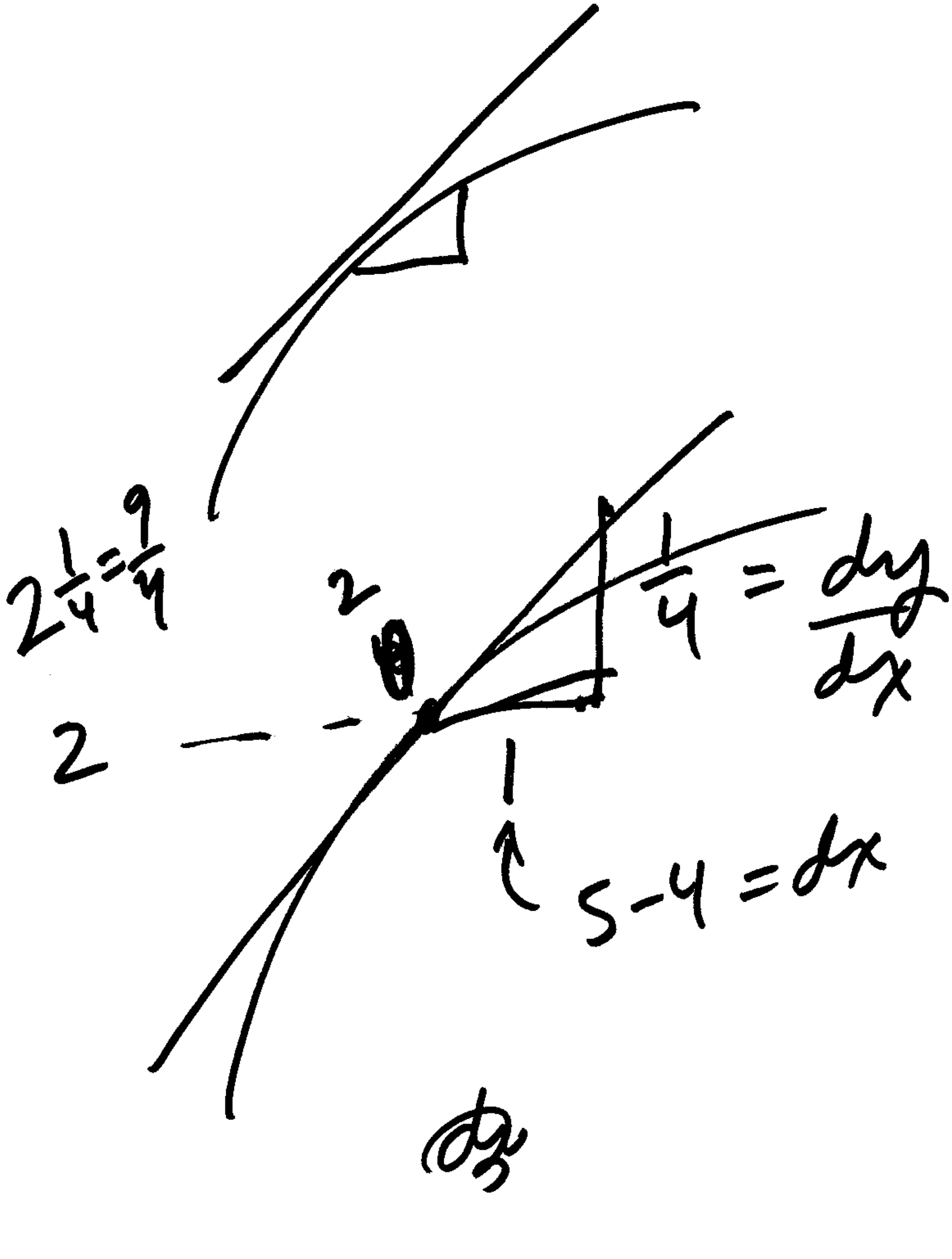
slope of tangent line $= f'(x) = \frac{dy}{dx}$. Thus $dy = f'(x)dx$.

Let $\Delta x = dx$. Then $\Delta y \sim dy$

$f(x + \Delta x) = f(x) + \Delta y \sim f(x) + dy$



$$\sqrt{5} \sim \frac{9}{4}$$



$2\frac{1}{4} = \frac{9}{4}$
2
2
$\frac{1}{4} = \frac{dy}{dx}$
1
$5-4 = dx$

Approx: $\sqrt{22}$

$$f(x) = \sqrt{x}$$

need value near 22
which I can evaluate

$$f(25) = \sqrt{25} = 5 \checkmark$$

Find tangent line at $x = 25$
and use that line to approx
$\sqrt{22}$

$$f'(x) = \frac{1}{2\sqrt{x}} = \cancel{}$$

$$f'(25) = \frac{1}{2 \cdot 5} = \frac{1}{10} = \text{slope}$$

point: $(25, 5)$

$$\frac{y-5}{x-25} = \frac{1}{10}$$

$$y - \cancel{5} = \frac{1}{10}(x-25) + 5$$

$$y = \frac{1}{10}x + \frac{25}{10} = g(x)$$

$$=$$

$$\sqrt{22} = f(22) \sim g(22)$$

$$= \frac{22}{10} + \frac{25}{10} = \frac{47}{10} = 4.7$$

Approx $\sin(0.1)$

$f(x) = \sin(x)$

Find tangent line to f at $x = 0$

$f'(x) = \cos(x)$

$f'(0) = 1 = $ slope

pt: $(0, f(0)) = (0, 0)$

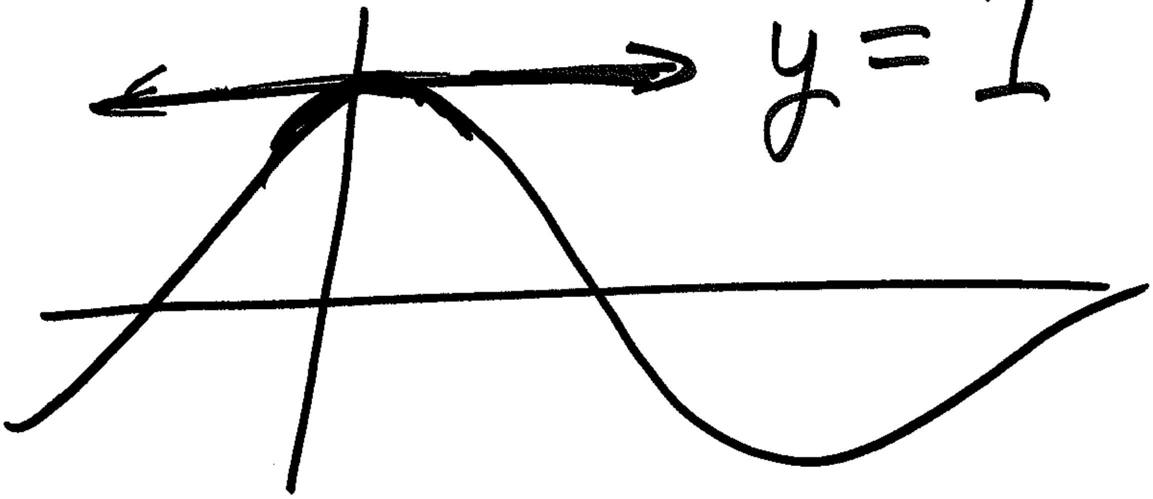
$y = x$ tangent line at $x=0$

$\sin(0.1) \sim 0.1$

For small x,

$\sin x \sim x$

Approx $\cos(0.1) \sim \underline{1}$



$$f(x) = \cos(x)$$
$$f'(x) = -\sin(x)$$
$$f'(0) = -\sin(0) = 0 = \text{slope}$$
$$pt = (0, f(0))$$
$$= (0, \cos(0)) = (0,1)$$

$$y = 1$$

For small x, $\cos x \sim \underline{1}$

Approx $\cos\left(\frac{3}{4}\right)$

$$f(x) = \cos(x)$$

near $x = \frac{\pi}{4}$

Find tangent line at $x = \frac{\pi}{4}$