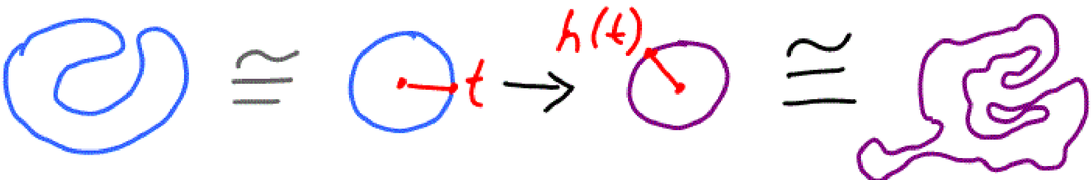
Feb 1, 2010: Chapter 2

Note Title 1/31/2010

Schonflies Thm: If J is a simple closed curve in S^2, then S^2 – J is the disjoint union of 2 disks.

Lemma (Alexander): Suppose A and B are homeomorphic to D^n. Any homeomorphism

$h: \partial A \to \partial B$ extends to a homeomorphism $h: A \to B$.

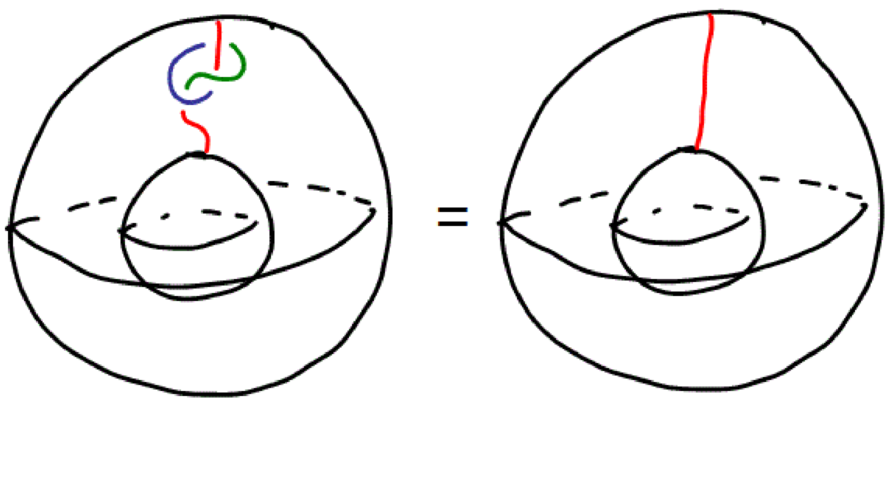


Annulus Thm: The closure of the region between two disjoint simple closed curves in S^2 is an annulus ($S^1 \times [0, 1]$).

Cor: Any two links in S^2 are equivalent.

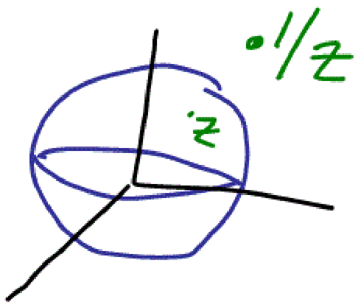
Cor: Any two knots in S^2 are ambient isotopic.

Knot theory in $S^2 \times S^1$ is different than in S^3



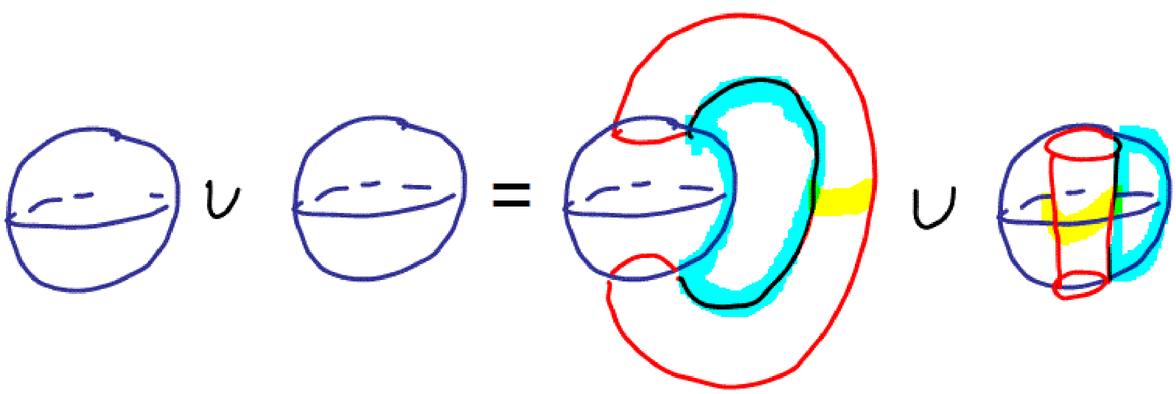
Recall

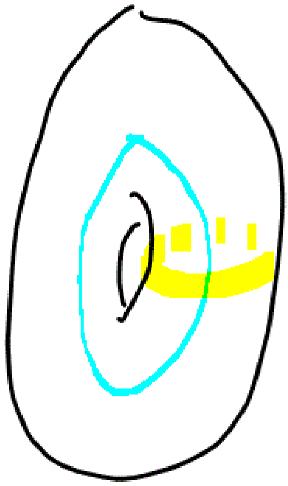
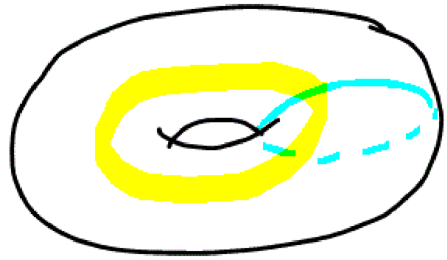
$$S^3 = \mathbb{R}^3 \cup \{\infty\} = B^3 \cup B^3$$

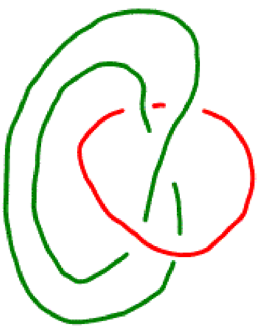
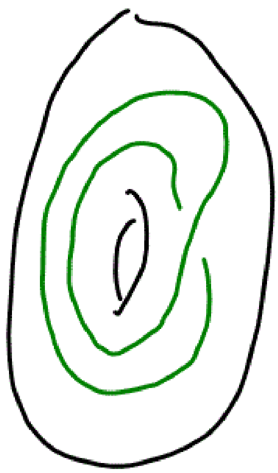
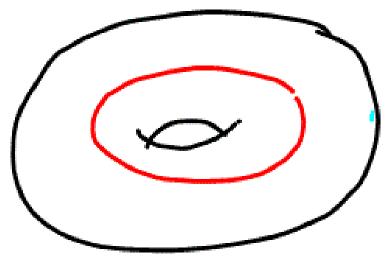


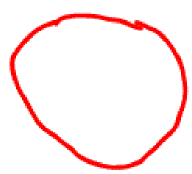
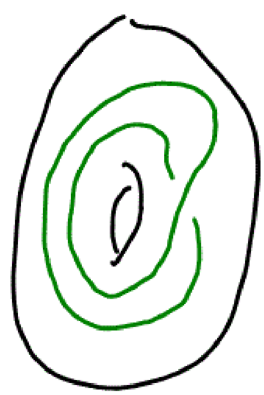
$$S^3 = B^3 \cup B^3 = V \cup V$$

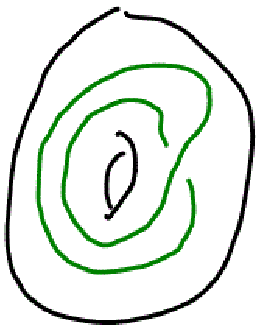
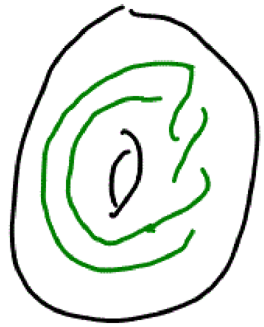
where V is a solid



$S^3 =$  $\cup$ 

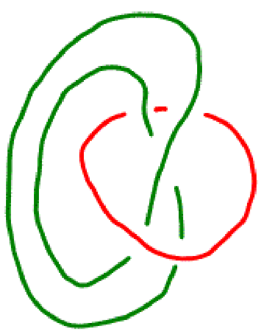
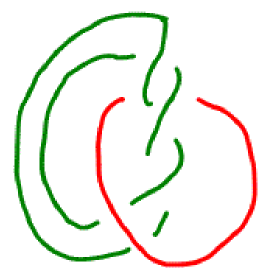
 $=$  $\cup$ 

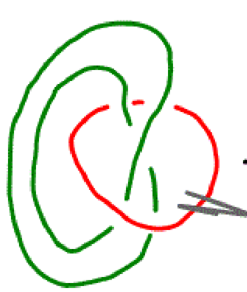
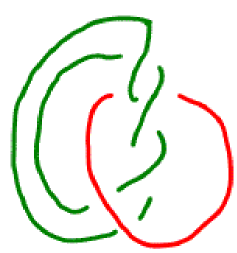
Thus $S^3 \setminus$  $=$ 

 $\cong$ 

I.e $\left(V, \right.$  $\left. \right) \cong \left(V, \right.$  $\left. \right)$

Hence

$S^3 \setminus$  $= S^3 \setminus$ 

But  $\neq$ 

Alternatively