Feb 3, 2010: Chapter 2

Note Title 2/3/2010

Goal: Classify knots in the torus.

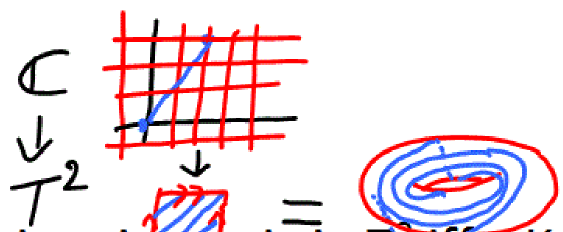
$\pi_1(T^2) = Z + Z$ since $T^2 = S^1 \times S^1$

<1, 0> = longitude = L
<0, 1> = meridian = M

Thus if f: $S^1 \rightarrow T^2$ is an embedding (ie a knot), then f is homotopic to <a, b> = aL + bM.

Thm 2C2: <a, b> is represented by an embedding iff a = b = 0 or gcd(a, b) = 1.

universal cover of the torus



Claim: K and K' are ambient isotopic in T^2 iff K = <a, b> = K' where a = b = 0 or gcd(a, b) = 1.

Proof (➔) Obvious.
(🡐) This direction requires several lemmas.

Recall K_1 and K_2 are ambient isotopic in M if there exists a map $h: M \times [0, 1] \to M$ such that

0.) h_t is a homeomorphism for all t in [0, 1] where $h_t: M \to M$, $h_t(x) = h(x, t)$.

1.) h_0 = identity

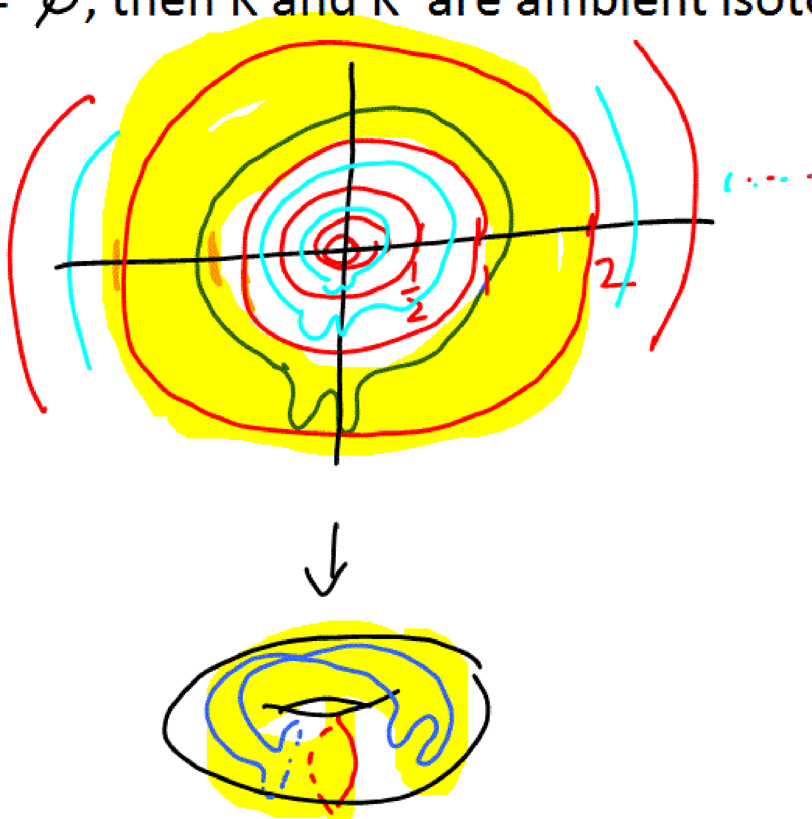
2.) $h_1(K_1) = K_2$

Recall K_1 and K_2 are homotopic in M if there exists a continuous map $h: S^1 \times [0, 1] \to M$ such that

1.) $h_0 = K_1$

2.) $h_1 = K_2$

Lemma 2C3: If K, K' of class <0, 1> and $K \cap K' = \emptyset$, then K and K' are ambient isotopic in T^2.

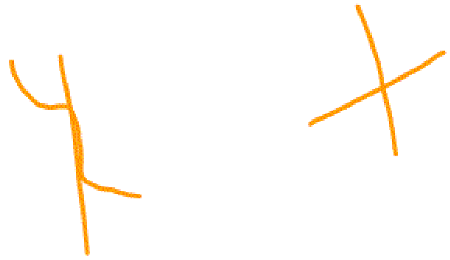


Lemma 2C5: If K of class <0, 1> such that K $\pitchfork$ M in a finite number of points, then K is ambient isotopic to M.

? $\cap$ transversely

Thm 2C8: Any two knots of class <0, 1> are ambient isotopic.

Proof: Exercise 2C9



2C10: Twist homeomorphisms

$h_L(e^{ix}, e^{iy}) = (e^{i(x+y)}, e^{iy})$ "longitudinal twist"

$h_M(e^{ix}, e^{iy}) = (e^{ix}, e^{i(x+y)})$ "meridinal twist"

Note $h_L(\langle 1, 0\rangle) = \langle 1, 0\rangle$, $h_L(\langle 0, 1\rangle) = \langle 1, 1\rangle$
$h_M(\langle 1, 0\rangle) = \langle 1, 1\rangle$, $h_M(\langle 0, 1\rangle) = \langle 0, 1\rangle$

Thm 2C16: 2 knots, $K, K' \subset T^2$ are ambient isotopic iff $[J] = [K]$

$\| $

$\langle a, b\rangle$

If K homotopic to $aL + bM$
$\gcd(a, b) = 1 \Rightarrow$

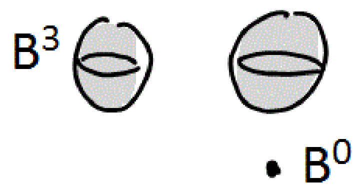
If $(a,b)=(0,0) \Rightarrow$ HW

Handle decomposition:

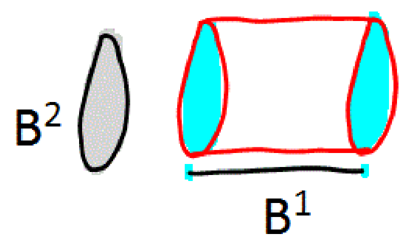
Let $B^n = \{ x \text{ in } R^n : ||x|| < 1\}$

A k-handle = $B^k \times B^{n-k}$ with gluing surface ∂B^k.

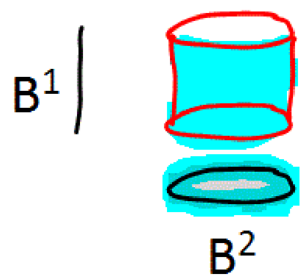
Ex: in R^3,



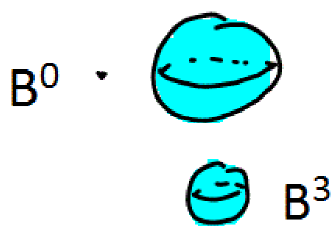
0-handle: $B^0 \times B^3$ = 3-ball



1-handle: $B^1 \times B^2$ = 3-ball



2-handle: $B^2 \times B^1$ = 3-



3-handle: $B^3 \times B^0$ = 3-ball