2.3 #23

$ma = F = -mg - \frac{v^2}{1325}$, $m = 0.15$, $g = 9.8$

$m\frac{dv}{dt} = -mg - \frac{v^2}{1325}$

$\frac{dv}{dt} = -g(1 + \frac{v^2}{1325mg})$

$\frac{dv}{1+\frac{v^2}{1325mg}} = -gdt$

$\int \frac{dv}{1+\frac{v^2}{1325mg}} = -g\int dt$

Let $u = \frac{v}{\sqrt{1325mg}}$, $du = \frac{dv}{\sqrt{1325mg}}$, $\sqrt{1325mg}\ du = dv$

$\int \frac{\sqrt{1325mg}du}{1+u^2} = -gt + C$

$\sqrt{1325mg}\ tan^{-1}(u) = -gt + C$

$\sqrt{1325mg}\ tan^{-1}(\frac{v}{\sqrt{1325mg}}) = -gt + C$

$t = 0, v = 20$

$\sqrt{1325mg}\ tan^{-1}(\frac{20}{\sqrt{1325mg}}) = C$

$> sqrt(1325 * .15 * 9.8);$

44.13332075

$> 44.1333 * arctan(20/44.1333);$

18.77823743

C = 18.77823743

$v = 0, t =?$

$\sqrt{1325mg}\ tan^{-1}(\frac{0}{\sqrt{1325mg}}) = -gt + C$

$0 = -gt + C,\ gt = C,\ t = \frac{C}{g}$

$> 18.77823743/9.8;$

1.916146677

$t = 1.916146677$

$$\sqrt{1325mg}\ tan^{-1}(\frac{v}{\sqrt{1325mg}}) = -gt + 18.77823743$$

$$tan^{-1}(\frac{v}{\sqrt{1325mg}}) = \frac{-gt+18.77823743}{44.13332075}$$

$$\frac{v}{\sqrt{1325mg}} = tan(\frac{-gt+18.77823743}{44.13332075})$$

$$v = 44.13332075\ tan(\frac{-gt+18.77823743}{44.13332075})$$

$$\frac{dx}{dt} = 44.13332075\ tan(\frac{-gt+18.77823743}{44.13332075})$$

$$dx = 44.13332075\ tan(\frac{-gt+18.77823743}{44.13332075})dt$$

$$dx = 44.13332075\frac{sin(\frac{-gt+18.77823743}{44.13332075})}{cos(\frac{-gt+18.77823743}{44.13332075})}dt$$

$$\text{Let } u = cos(\frac{-gt+18.77823743}{44.13332075})$$

$$du = -\frac{g}{44.13332075}sin(\frac{-gt+18.77823743}{44.13332075})dt$$

$$dx = -\frac{44.13332075^2}{g}\frac{du}{u}$$

$$\int dx = -\int \frac{44.13332075^2}{g}\frac{du}{u}$$

$$x = -\frac{44.13332075^2}{g}ln|u| + C$$

$x = -\frac{44.13332075^2}{g} ln|cos(\frac{-gt+18.77823743}{44.13332075})| + C$

$t = 0, x = 0(or 30)$

$> 44.1333^2/(-9.8) * ln(abs(cos(18.77823743/44.1333)));$

18.56151563

$C = 18.56151563$

$t = 1.91615, x = ?$

$x = -\frac{44.13332075^2}{9.8} ln|cos(\frac{-9.8(1.91615)+18.77823743}{44.13332075})| + C$

$> -44.1333^2/(9.8) * ln(abs(cos((-9.8 * 1.91615+ 18.77823743)/44.1333)));$

-0.

$x = 0 + 18.56151563 = 18.56151563$

Answer: 30 + 18.56151563 = 48.562 m

Short Method:

$m\frac{dv}{dt} = -mg - \frac{v^2}{1325}$

$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$

$mv\frac{dv}{dx} = -mg - \frac{v^2}{1325}$

$\frac{mvdv}{-mg-\frac{v^2}{1325}} = dx$

Let $u = -mg - \frac{v^2}{1325}$, $du = -\frac{2vdv}{1325}$,

$-\frac{1325mdu}{2u} = dx$

$-\int \frac{1325mdu}{2u} = \int dx$

$-\frac{1325m}{2}ln|u| = x + C$

$-\frac{1325m}{2}ln|-mg - \frac{v^2}{1325}| = x + C$

$-\frac{1325m}{2}ln|mg + \frac{v^2}{1325}| = x + C$

$x = 0, v = 20$

$-\frac{1325m}{2} ln|mg + \frac{20^2}{1325}| = C$

$> -1325 * .15/2 * ln(.15 * 9.8 + 20^2/1325);$

-56.84696820

$C = -56.84696820$

$v = 0, x = ?$

$-\frac{1325m}{2} ln|mg| - C = x$

$> -1325 * .15/2 * ln(.15 * 9.8);$

-38.28545108

$> -38.28545108 - (-56.84696820);$

18.56151712

Answer: 30 + 18.56151563 = 48.562 m