$mu'' + \gamma u' + ku = F_0 cos(\omega t)$

Non-homogeneous solution: Guess $\phi(t) = Acos(\omega t) + Bsin(\omega t)$.

$\phi'(t) = -A\omega sin(\omega t) + B\omega cos(\omega t)$.

$\phi''(t) = -A\omega^2 cos(\omega t) - B\omega^2 sin(\omega t)$.

$mu'' + \gamma u' + ku = F_0 cos(\omega t)$

$$m[-A\omega^2 cos(\omega t) - B\omega^2 sin(\omega t)] + \gamma[-A\omega sin(\omega t) + B\omega cos(\omega t)] + k[Acos(\omega t) + Bsin(\omega t)] = F_0 cos(\omega t)$$

$cos(\omega t)[-mA\omega^2 + \ gB\omega + kA] + sin(\omega t)[-mB\omega^2 - \gamma A\omega + kB] = F_0 cos(\omega t)$

$cos(\omega t)[(-m\omega^2 + k)A + \ g\omega B] + sin(\omega t)[(-m\omega^2 + k)B - \gamma\omega A] = F_0 cos(\omega t)$

$(-m\omega^2 + k)A + \gamma\omega B = F_0$

$(-m\omega^2 + k)B - \gamma\omega A = 0$. Thus $A = \frac{(-m\omega^2+k)B}{\gamma\omega}$

Hence $\frac{(-m\omega^2+k)^2 B}{\gamma\omega} + \gamma\omega B = F_0$

$(-m\omega^2 + k)^2 B + (\gamma\omega)^2 B = F_0\gamma\omega$

$[(-m\omega^2 + k)^2 + (\gamma\omega)^2]B = F_0\gamma\omega$

$B = \frac{F_0\gamma\omega}{[(-m\omega^2+k)^2+(\gamma\omega)^2]}$

Thus $A = \frac{(-m\omega^2+k)F_0\gamma\omega}{\gamma\omega[(-m\omega^2+k)^2+(\gamma\omega)^2]} = \frac{(-m\omega^2+k)F_0}{[(-m\omega^2+k)^2+(\gamma\omega)^2]}$

$\phi(t) = Acos(\omega t) + Bsin(\omega t).$

Let $A = Rcos(\delta)$, $B = Rsin(\delta)$ in $Acos(\omega t) + Bsin(\omega t)$. Thus,

$Acos(\omega t) + Bsin(\omega t) = Rcos(\delta)cos(\omega t) + Rsin(\delta)sin(\omega t) = Rcos(\omega t - \delta)$

where $R = \sqrt{A^2 + B^2} = \sqrt{(\frac{(-m\omega^2+k)F_0}{[(-m\omega^2+k)^2+(\gamma\omega)^2]})^2 + (\frac{F_0\gamma\omega}{[(-m\omega^2+k)^2+(\gamma\omega)^2]})^2}$

$= \frac{F_0}{[(-m\omega^2+k)^2+(\gamma\omega)^2]}\sqrt{((-m\omega^2 + k))^2 + (\gamma\omega)^2}$

$= \frac{F_0}{\sqrt{((-m\omega^2+k))^2+(\gamma\omega)^2}} = \frac{F_0}{\sqrt{(m(\frac{k}{m}-\omega^2))^2+(\gamma\omega)^2}}$

So $R = \frac{F_0}{\sqrt{m^2({\omega_o}^2-\omega^2)^2+(\gamma\omega)^2}}$ for ${\omega_o}^2 = \frac{k}{m}$

$tan\delta = \frac{B}{A}$

$cos\delta = \frac{A}{R} = \frac{m(\omega_{)}^2-\omega^2}{\sqrt{m^2({\omega_o}^2-\omega^2)^2+(\gamma\omega)^2}}$, $\qquad sin\delta = \frac{B}{R} = \frac{\gamma\omega}{\sqrt{m^2({\omega_o}^2-\omega^2)^2+(\gamma\omega)^2}}$