

1.1:

Examples of differentiable equation:

1.) $F = ma = m \frac{dv}{dt} = mg - \gamma v$

2.) Mouse population increases at a rate proportional to the current population:

More general model : $\frac{dp}{dt} = rp - k$

where r = growth rate or rate constant,

k = predation rate = # mice killed per unit time.

direction field = slope field = graph of $\frac{dv}{dt}$ in t, v -plane.

*** can use slope field to determine behavior of v including as $t \rightarrow \infty$.

Equilibrium Solution = constant solution

1.2:

Solved $\frac{dy}{dt} = ay + b$ by separating variables:

$$\frac{dy}{ay+b} = dt$$

$$\int \frac{dy}{ay+b} = \int dt$$

$$\frac{\ln|ay+b|}{a} = t + C$$

$$\ln|ay + b| = at + C$$

$$e^{\ln|ay+b|} = e^{at+C}$$

$$|ay + b| = e^C e^{at}$$

$$ay + b = \pm(e^C e^{at})$$

$$ay = Ce^{at} - b$$

$$y = Ce^{at} - \frac{b}{a}$$

Initial Value Problem: $y(t_0) = y_0$

1.3:

ODE (ordinary differential equation): single independent variable

$$\text{Ex: } \frac{dy}{dt} = ay + b$$

vs

PDE (partial differential equation): several independent variables

$$\text{Ex: } \frac{\partial xy}{\partial x} = \frac{\partial xy}{\partial y}$$

order of differential eq'n: order of highest derivative

example of order n : $y^{(n)} = f(t, y, \dots, y^{(n-1)})$

Linear vs Non-linear

linear: $a_0(t)y^{(n)} + \dots + a_n(t)y = g(t)$

$$\text{Ex: } ty'' - t^3y' - 3y = \sin(t)$$

$$\text{Ex: } 2y'' - 3y' - 3y^2 = 0$$

*****Existence of a solution*****

*****Uniqueness of solution*****

CH 2: Solve $\frac{dy}{dt} = f(t, y)$

2.1: First order linear eqn: $\frac{dy}{dt} + p(t)y = g(t)$

Ex 1: $y' = ay + b$

Ex 2: $y' + 3t^2y = t^2, y(0) = 0$

Note: could use section 2.2 method, separation of variables to solve ex 1 and 2.

Ex 3: $t^2y' + 2ty = t\sin(t)$

Ex 1: $2\frac{dy}{dt} + 10y = 16$