Graph theory problem called find ~~perfect~~ optimal perfect matching

Chinese Postman Problem: Find shortest (or lowest weight) closed walk that visits all edges (note some edges may be repeated).

Find optimal Eulerization: ← solution

1.) Find all odd degree vertices (note: ∃ an even number of odd degree vertices).

2.) For each possible pairing (perfect matching) of odd degree vertices, v_i, v_j,

dijkstra's alg

Find the shortest path or minimum weight path, $P_{i,j}$, between each pair v_i, v_j.

3.) Take optimal pairing (perfect matching), and duplicate each edge in the path $P_{i,j}$ for each i, j in the optimal pairing. Note this creates an Eulerian graph G'

4.) Find Eulerian circuit in the Eulerian graph G'.

5.) Apply to the original graph by repeating edges that were duplicated to create the Eulerian graph G'.

look at all possible pairings

Konigsberg bridge with made up distances.

Find shortest tour that visits each bridge and returns to starting point.

I.e. Find lowest weight closed walk.
Thus find optimal Eulerization.

Note all vertices have odd degree.

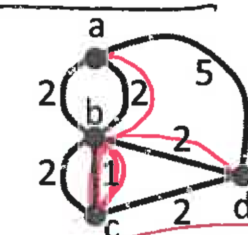
~~duplicat~~ duplicate

pair: bc $b\langle b\,c\rangle c$ +1

pair: ad

$a\langle ab\rangle b\langle bd\rangle d$

+2 + 2 = +4

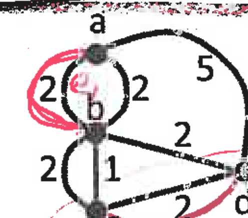


+1+~~5~~ = +5

pair: ab a, e_1, b +2

pair: cd $c\langle c,d\rangle d$ +2

+2+2 = +4



optimal

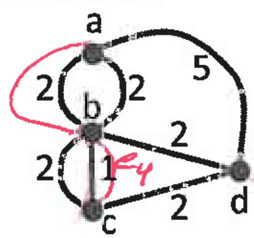
pair: ac $a\, e_1\, b\, e_4\, c$

+2 +1 = +3

pair: bd $b\langle b,d\rangle d$

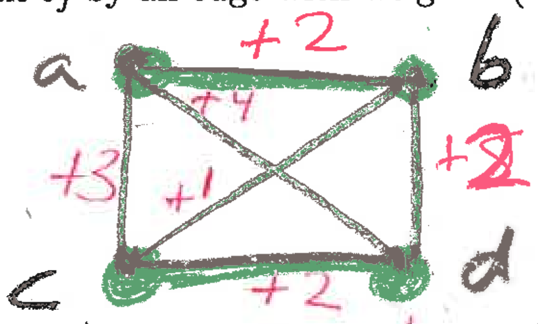
+2

+2+3 = +5



all possible pairings = Complete graph

Algorithm 4.2 step 2: Construct a weighted complete graph on $2k$ vertices in which vertex v_i is joined to vertex v_j by an edge with weight $w(P_{i,j})$:



M = {<a,b>, <c,d>}

Find an optimal perfect matching. ← Graph theory problem

Marriage Problem

Defn: A subset, M, of edges of a gragh G is a *matching* if $e, e' \in M$, then e, e' are not incident to the same vertex.

Defn: A matching is *perfect* if every vertex in G is incident to some edge in M.

Some Applications of Matching:

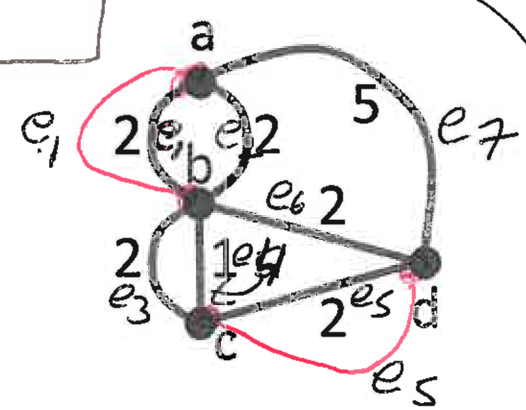
Marriage.

Assignment of tasks to individuals where one task is assigned to each individual.

Eulerizing a graph.

perfect matching

Textbooks often call perfect optimal matching the marriage problem



Find shortest tour that visits each bridge and returns to starting point. I.e. Find lowest weight closed walk.

I.e. Find lowest weight closed walk.

a e1 b e1 a e2, b, e3 c, e4 b, e6, d, e5 c, e5 d, e7, a

If all vertices have even degree, then lowest weight closed walk = Eulerian circuit.

If exactly 2 vertices, u, w, have odd degree, apply Dijkstra's algorithm to u (or w).

If $2k$ vertices have odd degree, form complete graph on $2k$ vertices and use Dijkstra's algorithm $2k - 1$ times to determine edge weights. Then find optimal pairing.

Sol'ns to Chinese Postman Problem