Homework Solutions

1. $(2, 3, 3)$ On the board

2. $(2, 4, 1, 3)$ On the board

3. $3 + \dfrac{1}{3 + \frac{1}{2}} = 3 + \dfrac{1}{\frac{7}{2}} = 3 + \dfrac{2}{7} = \dfrac{23}{7}$

4. $3 + \dfrac{1}{1 + \frac{1}{4 + \frac{1}{2}}} = 3 + \dfrac{1}{1 + \frac{1}{\frac{9}{2}}} = 3 + \dfrac{1}{1 + \frac{2}{9}} = 3 + \dfrac{1}{\frac{11}{9}} = 3 + \dfrac{9}{11} = \dfrac{42}{11}$

5. $3 + \dfrac{1}{-4 + \frac{1}{2}} = 3 + \dfrac{1}{\frac{-7}{2}} = 3 + \dfrac{-2}{7} = \dfrac{19}{7}$

6. Yes. There are two approaches to solving this problem. Since (2,-4,3) was already calculated we can do the following:

   (a) Find the fraction associated to $(1, 1, 2, 1, 2)$ and compare.
   $$2 + \dfrac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1}}}} = 2 + \dfrac{1}{1 + \frac{1}{2 + \frac{1}{2}}} = 2 + \dfrac{1}{1 + \frac{1}{\frac{5}{2}}} = 2 + \dfrac{1}{1 + \frac{2}{5}} = 2 + \dfrac{1}{\frac{7}{5}} = 2 + \dfrac{5}{7} = \dfrac{19}{7}$$
   or,

   (b) Find the canonical form of the rational tangle associated to $\dfrac{19}{7} \Rightarrow$
   $$\dfrac{19}{7} = 2 + \dfrac{5}{7} = 2 + \dfrac{1}{\frac{7}{5}} = 2 + \dfrac{1}{1 + \frac{2}{5}} = 2 + \dfrac{1}{1 + \frac{1}{\frac{5}{2}}} = 2 + \dfrac{1}{1 + \frac{1}{2 + \frac{1}{2}}} = 2 + \dfrac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1}}}}$$

7. $\dfrac{9}{7} = 1 + \dfrac{2}{7} = 1 + \dfrac{1}{\frac{7}{2}} = 1 + \dfrac{1}{3 + \frac{1}{2}} \Rightarrow (2, 3, 1)$

8. $N(\frac{9}{7}) = 6_1^*$

9. (a) $N(-2, 1) = N(1 + \frac{1}{-2}) = N(\frac{1}{2}) \equiv N(1) \to 0_1$ knot.
   (b) $N(-2, 2) = N(2 + \frac{1}{-2}) = N(\frac{3}{2}) \to 3_1^*$ knot.
   (c) $N(-2, 3) = N(3 + \frac{1}{-2}) = N(\frac{5}{2}) \equiv N(\frac{5}{3}) \to 4_1$ knot.