1.3 Vectors in  $\mathbb{R}^m$ 

Defn:  $\mathbf{u} = (u_1, ..., u_m), \ \mathbf{v} = (v_1, ..., v_m)$  are **vectors** in  $\mathbf{R}^{\mathbf{m}}$ .

Defn:  $u_1, ..., u_m$  are the **components** of **u**.

Defn:  $\mathbf{u} = \mathbf{v}$  if and only if  $u_i = v_i$  for all *i*.

Defn: The **zero vector** in  $\mathbf{R}^{\mathbf{m}}$  is the m-vector  $\mathbf{0} = (0, 0, ..., 0)$ .

Vector Addition

Defn: The sum of **u** and **v** is the vector  $\mathbf{u} + \mathbf{v} = (u_1 + v_1, ..., u_m + v_m).$ 

Defn: The **negative** of **u** is the vector  $-\mathbf{u} = (-u_1, ..., -u_m)$ 

Defn: The **difference** between **u** and **v** is the vector  $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = (u_1 - v_1, ..., u_m - v_m).$ 

Defn: In this class a scalar, c, is a real number.

Defn: The scalar multiple of  $\mathbf{u}$  by c is the vector  $c\mathbf{u} = (cu_1, ..., cu_m)$ .

Thm: The vectors,  $\mathbf{u}$  and  $\mathbf{v}$ , are collinear iff there exists a scalar c such that  $\mathbf{v} = c\mathbf{u}$ . In this case

a.) if c > 0, **u** and c**u** have the same direction.

b.) If c < 0, **u** and c**u** have opposite directions.

Defn: The *length (norm, magnitude)* of  $\mathbf{u}$  is its distance from  $\mathbf{0}$  and is denoted by

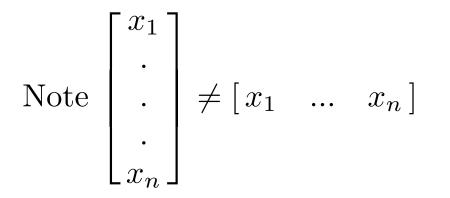
$$||\mathbf{u}|| = d(\mathbf{0}, \mathbf{u}) = \sqrt{u_1^2 + u_2^2 + \dots + u_m^2}.$$

Two vectors are equivalent if they have the same direction and length.

Parallelogram rule:

Addition: the directed line segment starting at  $\mathbf{u}$  and ending at  $\mathbf{u} + \mathbf{v}$  is equivalent to  $\mathbf{v}$ 

Subtraction: the directed line segment starting at  $\mathbf{u}$  and ending at  $\mathbf{v}$  is equivalent to  $\mathbf{v} - \mathbf{u}$ 



However, we will sometimes abuse notation.

Thm 3.2.1 (or thm 4.1.1 p163)  
a.) 
$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$
  
b.)  $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$   
c.)  $\mathbf{u} + \mathbf{0} = \mathbf{u}$   
d.)  $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$   
e.)  $(cd)\mathbf{u} = c(d\mathbf{u})$   
f.)  $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$   
g.)  $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$   
h.)  $1\mathbf{u} = \mathbf{u}$ 

Sometimes we will write the vector  $\mathbf{x}$  as a row vector:  $(x_1, ..., x_n)$ .

Other times we will write the vector  $\mathbf{x}$  as a column vector:

$$\begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$