1.3 Vectors in R^m

Defn: $\mathbf{u} = (u_1, ..., u_m)$, $\mathbf{v} = (v_1, ..., v_m)$ are **vectors** in $\mathbf{R^m}$.

Defn: $u_1, ..., u_m$ are the **components** of $\mathbf{u}$.

Defn: $\mathbf{u} = \mathbf{v}$ if and only if $u_i = v_i$ for all i.

Defn: The **zero vector** in $\mathbf{R^m}$ is the m-vector $\mathbf{0} = (0, 0, ..., 0)$.

Vector Addition

Defn: The **sum** of $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} + \mathbf{v} = (u_1 + v_1, ..., u_m + v_m)$.

Defn: The **negative** of $\mathbf{u}$ is the vector $-\mathbf{u} = (-u_1, ..., -u_m)$

Defn: The **difference** between $\mathbf{u}$ and $\mathbf{v}$ is the vector $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = (u_1 - v_1, ..., u_m - v_m)$.

Defn: In this class a **scalar**, c, is a real number.

Defn: The **scalar multiple** of $\mathbf{u}$ by c is the vector $c\mathbf{u} = (cu_1, ..., cu_m)$.

Thm: The vectors, $\mathbf{u}$ and $\mathbf{v}$, are collinear iff there exists a scalar c such that $\mathbf{v} = c\mathbf{u}$. In this case

a.) if $c > 0$, $\mathbf{u}$ and $c\mathbf{u}$ have the same direction.

b.) If $c < 0$, $\mathbf{u}$ and $c\mathbf{u}$ have opposite directions.

Defn: The *length (norm, magnitude)* of $\mathbf{u}$ is its distance from $\mathbf{0}$ and is denoted by

$$||\mathbf{u}|| = d(\mathbf{0}, \mathbf{u}) = \sqrt{u_1^2 + u_2^2 + ... + u_m^2}.$$

Two vectors are equivalent if they have the same direction and length.

Parallelogram rule:

Addition: the directed line segment starting at $\mathbf{u}$ and ending at $\mathbf{u}+\mathbf{v}$ is equivalent to $\mathbf{v}$

Subtraction: the directed line segment starting at $\mathbf{u}$ and ending at $\mathbf{v}$ is equivalent to $\mathbf{v}-\mathbf{u}$

Note $\begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix} \neq [\, x_1 \quad \ldots \quad x_n \,]$

However, we will sometimes abuse notation.

Thm 3.2.1 (or thm 4.1.1 p163)

a.) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

b.) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

c.) $\mathbf{u} + \mathbf{0} = \mathbf{u}$

d.) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

e.) $(cd)\mathbf{u} = c(d\mathbf{u})$

f.) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

g.) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

h.) $1\mathbf{u} = \mathbf{u}$

Sometimes we will write the vector $\mathbf{x}$ as a row vector: $(x_1, ..., x_n)$.

Other times we will write the vector $\mathbf{x}$ as a column vector: $\begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_n \end{bmatrix}$