Math 2418 Linear Algebra Exam #2 Name: ____________________

October 31, 2001 SHOW ALL WORK Circle one: Wednesday/Thursday

[10] 1a.) Find the standard matrix for the linear transformation T defined by the formula $T((x_1, x_2)) = (5x_1 + 3x_2,\ x_1 - 4x_2,\ x_2,\ 0)$.

Answer 1a.) ____,

[2] 1b.) The domain of T is __________.

[2] 1c.) The codomain of T is __________.

[2] 1d.) Is T one-to-one?__________

[2] 1e.) Is T onto?__________

[2] 1f.) $T([\mathbf{e_1}]) =$

[3] 1g.) Find three vectors which are in the image of T.

Answer 1g.) ____

[3] 1h.) Find two vectors in the codomain of T which are NOT in the image of T.

Answer 1h.) ____

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2.) Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 1 & 0 & 2 & 3 & 0 \\ 1 & 0 & 5 & 9 & 3 \end{bmatrix}$.

[6] 2a.) Find a basis for the column space of A.

Answer 2a.) ____________________

[6] 2b.) Find a basis for the row space of A.

Answer 2b.) ____________________

[8] 2c.) Find a basis for the nullspace of A.

Answer 2c.) ____________________

[3] 2d.) Rank(A) = ________.

[3] 2e.) Nullity(A) = ________.

[3] 2f.) Write one of the columns of A as a linear combination of the other columns of A.

Answer 2f.) ____________________

[2] 2g.) Solve Ax = 0.

Answer 2g.) ____________________

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[12] 3a.) Write as $1 - 8t + 5t^2$ as a linear combination of $1 + 4t - 3t^2$ and $2 + 5t - 4t^2$.

Answer 3a.) $1-8t+5t^2 =$ ____

[3] 3b.) Is $\{1 + 4t - 3t^2,\ 2 + 5t - 4t^2\}$ a basis for $span\{1 + 4t - 3t^2,\ 2 + 5t - 4t^2\}$? ________.

[3] 3c.) Is $\{1 + 4t - 3t^2,\ 2 + 5t - 4t^2\}$ a basis for P^2? ________.

[2] 3d.) Is t^2 in the span of $\{1 + 4t - 3t^2,\ 2 + 5t - 4t^2\}$? ________.

[2] 3e.) Is $\{1 + 4t - 3t^2,\ 2 + 5t - 4t^2,\ t^2\}$ a basis for P^2? ________.

[4] 4a.) $(2, 0, 1, -3) \cdot (5, 4, 8, 2) =$ ________.

[3] 4b.) Is $(2, 0, 1, -3)$ orthogonal to $(5, 4, 8, 2)$? ________.

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5.) Circle the correct answer and fill in the blank if appropriate.

[3] 5a.) Suppose that A is a 4×4 matrix, Rank$(A) = 4$, and Rank$[A|\mathbf{b}] = 4$. Then $A\mathbf{x} = \mathbf{b}$ has

i.) no solution

ii) exactly one solution

iii) infinite number of solutions with ________ parameters (free variables).

[3] 5b.) Suppose that A is a 4×7 matrix, Rank$(A) = 4$, and Rank$[A|\mathbf{b}] = 4$. Then $A\mathbf{x} = \mathbf{b}$ has

i.) no solution

ii) exactly one solution

iii) infinite number of solutions with ________ parameters.

[3] 5c.) Suppose that A is a 4×7 matrix, Rank$(A) = 3$, and Rank$[A|\mathbf{b}] = 4$. Then $A\mathbf{x} = \mathbf{b}$ has

i.) no solution

ii) exactly one solution

iii) infinite number of solutions with ________ parameters.

[3] 5d.) Suppose that A is a 3×3 matrix whose column space is a line through the origin. Then the nullspace of A is

i.) the empty set.

ii.) a point.

iii.) a line.

iv.) a 2-dimensional plane.

v.) a 3-dimensional hyperplane.

[5] 6.) Give an example of a 3×3 matrix whose column space is a line through the origin.