Thm: Let $A = (a_{ij})$ by an $n \times n$ square matrix, $n > 1$. Then expanding along row i,

$$\boxed{detA = \Sigma_{k=1}^{n}(-1)^{i+k} a_{ik} detA_{ik}.}$$

Or expanding along column j,

$$\boxed{detA = \Sigma_{k=1}^{n}(-1)^{k+j} a_{kj} detA_{kj}.}$$

Choose row or column w/ most 0's

Defn: $detA_{ij}$ is the i, j-minor of A.

$(-1)^{i+j} detA_{ij}$ is the i, j-cofactor of A.

OR

3.2: Properties of Determinants

Thm: If $A \xrightarrow{R_i \to cR_i} B$, then $detB = c(detA)$.

Warning note: $det(cA) = c^n detA$.

Thm: If $A \xrightarrow{R_i \leftrightarrow R_j} B$, then $detB = -(detA)$.

Thm: If $A \xrightarrow{R_i + cR_j \to R_i} B$, then $detB = detA$.

$\underline{1} R_i + cR_j \to \underline{1} R_i$

CREATE ZEROS

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \xrightarrow[R_1 \to 2R_1]{\cancel{2R_1 \to R_1}} \begin{vmatrix} 2a & 2b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} 2a & 2b \\ c & d \end{vmatrix} = 2ad - 2bc = 2(ad - bc)$$

$$2 \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} 2a & 2b \\ 2c & 2d \end{vmatrix}$$

$$= 4(ad - bc)$$

$$= 2^2(ad - bc$$

1

$$A \xleftrightarrow{R_i \leftrightarrow R_j} B$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{vmatrix} c & d \\ a & b \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} c & d \\ a & b \end{vmatrix} = cb - ad = -(ad-bc)$$

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 8 \end{matrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{matrix} 1 & 2 & 3 \\ \cancel{7} & \cancel{0} & \cancel{8} \\ 4 & 5 & 6 \end{matrix} \qquad \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$-7\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} + 0\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} - 8\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

Compare to 3/3 notes p. 3

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 8 \end{matrix} \xrightarrow[R_1 \leftrightarrow R_3]{} \begin{matrix} 7 & 0 & 8 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{matrix}$$

$$+7\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} - 0\begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + 8\begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix}$$

$$= -(-45) = +45$$

compare to 3/3 notes p. 3

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \xrightarrow{R_2 - 2R_1 \to R_2} \begin{vmatrix} a & b \\ c-2a & d-2b \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b \\ c-2a & d-2b \end{vmatrix} = a(d-2b) - b(c-2a)$$

$$= ad - \cancel{2ab} - bc + \cancel{2ab}$$

$$= ad - bc$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \xrightarrow{R_1 - 2R_2 \to R_2} \begin{vmatrix} a & b \\ a-2c & b-2d \end{vmatrix}$$

$$= a(b-2d) - b(a-2c)$$

$$= \cancel{ab} - 2ad - \cancel{ba} + 2bc$$

$$= -2(ad - bc)$$

4

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \xrightarrow[-2R_2 \to R_2]{} \begin{vmatrix} a & b \\ -2c & -2d \end{vmatrix}$$

affects the determinate
$\times(-2)$

$$\Big\downarrow R_1 + R_2 \to R_2$$

$$\begin{vmatrix} a & b \\ a-2c & b-2d \end{vmatrix}$$

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$$\begin{vmatrix} 2 & 1 & 3 & 2 \\ 2^{-2} & 1^{-1} & -1^{-3} & 1^{-2} \\ 0 & 3 & -2 & 1 \\ 4^{-4} & 1^{-2} & 2^{-6} & 2^{-4} \end{vmatrix}$$

$$\left.\begin{array}{l} 1R_2 - R_1 \rightarrow 1R_2 \\ R_4 - 2R_1 \rightarrow R_4 \end{array}\right\} \text{determinant does not change}$$

$$\begin{vmatrix} 2 & 1 & 3 & 2 \\ 0 & 0 & -4 & -1 \\ 0 & 3 & -2 & 1 \\ 0 & -1 & -4 & -2 \end{vmatrix}$$

$$R_2 \leftrightarrow R_4 \qquad R_2 \leftrightarrow R_4$$

$$-\begin{vmatrix} 2 & 1 & 3 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & 3^{-3} & -2^{-12} & 1^{-6} \\ 0 & 0 & -4 & -1 \end{vmatrix} \longrightarrow R_3 + 3R_2 \rightarrow R_3$$

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$$= - \begin{vmatrix} 2 & 1 & 3 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & 0 & -14 & -5 \\ 0 & 0 & -4 & -1 \end{vmatrix}$$

$$\Big\downarrow \; R_3/{-14} \rightarrow R_3$$

$$= \frac{-1}{-14} \begin{vmatrix} 2 & 1 & 3 & 2 \\ 0 & -1 & -4 & -2 \\ 0 & 0 & 1 & 5/14 \\ 0 & 0 & -4 & -1 \end{vmatrix}$$