Math 2550 Matrix Algebra Exam #1 (Form D)
Feb. 26, 2014 SHOW ALL WORK

[30] 1.) Solve the following systems of equations. Write your answer in parametric vector format (note this is a multipart question).

1a.) $$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|ccc} 2 & -4 & -36 & 26 & -12 & 0 & 4 & 30 \\ 7 & -1 & 4 & 0 & 2 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \\ 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 2 & -4 & -36 & 26 & -12 & 0 & 4 & 30 \\ 7 & -1 & 4 & 0 & 2 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 0 & -4 & -40 & 28 & -14 & 0 & 2 & 30 \\ 0 & -1 & -10 & 14 & -5 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & -7 & 5 & 0 & 1 & 0 \\ 0 & -4 & -40 & 28 & -14 & 0 & 2 & 30 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & -7 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 6 & 30 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & -7 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 & 6 & 0 & 6 & 30 \end{array}\right]$$

$$\left[\begin{array}{ccccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 10 & -7 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \end{array}\right]$$

We now have echelon form.

Note last equation for 1b.) $0 = 6$, thus no solution. Thus we can drop the system corresponding to problem 1b as we already know the anwer for 1b) no solution.

Working from right to left to get REF:

$$\left[\begin{array}{ccccc|cc} 1 & 0 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 10 & -7 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccccc|cc} 1 & 0 & 2 & -1 & 0 & 0 & -5 \\ 0 & 1 & 10 & -7 & 0 & 0 & -25 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\text{Thus } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_3 + x_4 \\ -10x_3 + 7x_4 \\ x_3 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -10 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 7 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4$$

1a.) $$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Answer: } \underline{\begin{bmatrix} -2 \\ -10 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 7 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4}$$

1b.) $$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

Answer: $\underline{\ NoSolution\ }$

1c.) $$\begin{bmatrix} 2 & -4 & -36 & 26 & -12 \\ 7 & -1 & 4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 \\ 1 & 0 & 2 & -1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$$

$$\text{Answer: } \underline{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ -10 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ 7 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -5 \\ -25 \\ 0 \\ 0 \\ 5 \end{bmatrix}}$$

[20] 2.) Let A be the coefficient matrix from problem 1.

2a.) Is $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ in the span of the columns of A? ___*yes*___

If so, write $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of the columns of A:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} 2 \\ 7 \\ 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} -4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} -36 \\ 4 \\ 0 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} 26 \\ 0 \\ 0 \\ -1 \end{bmatrix} + 0 \begin{bmatrix} -12 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

2b.) Is $\begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$ in the span of the columns of A? ___*no*___

If so, write $\begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$ as a linear combination of the columns of A:

2c.) Is $\begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$ in the span of the columns of A? ___*yes*___

If so, write $\begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix}$ as a linear combination of the columns of A:

$$\begin{bmatrix} 30 \\ 0 \\ 15 \\ 0 \end{bmatrix} = \text{-5} \begin{bmatrix} 2 \\ 7 \\ 0 \\ 1 \end{bmatrix} + -25 \begin{bmatrix} -4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} -36 \\ 4 \\ 0 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} 26 \\ 0 \\ 0 \\ -1 \end{bmatrix} + 5 \begin{bmatrix} -12 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

3.) Let A be the coefficient matrix from problem 1.

[4] 3a.) Are the columns of A linearly independent? <u>*no*</u>

[4] 3b.) Do the columns of A span R^4? <u>*no*</u>

[1 point extra credit] 3c.) Given an example of 4 vectors that span R^4 where 3 of your vectors are columns of A. Briefly explain.

$$\begin{bmatrix} 2 \\ 7 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -12 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

By work on page 1, if I form a matrix from these 4 vectors, the echelon form of this matrix will have a pivot in all 4 rows. Thus $Cx = b$ will have a solution for all b in R^4 if the columns of C consist of the 4 vectors above.

[10] 4a.) Find the inverse of $\begin{bmatrix} 3 & 12 \\ 2 & 10 \end{bmatrix}$

$$\begin{bmatrix} 3 & 12 \\ 2 & 10 \end{bmatrix} \begin{bmatrix} 10 & -12 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Answer: $\underline{\begin{bmatrix} \frac{5}{3} & -2 \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix}}$

[4] 4b.) Use the inverse found in part a to solve the following system of equations:

3x + 12 y = 0
2x + 10y = 1

$$\begin{bmatrix} 3 & 12 \\ 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{5}{3} & -2 \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & 12 \\ 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & -2 \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{1}{2} \end{bmatrix}$$

Answer: $\underline{\begin{bmatrix} -2 \\ \frac{1}{2} \end{bmatrix}}$

[5] 5.) Given an example of matrices A and B where neither are the zero matrix, but $AB = 0$.

There are many possible examples such as $\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [(1)(0) + (0)(1)] = [0]$

Answer: $A = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Or $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Answer: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Note there are many, many potential answers. I choose one with lots of zeros since I knew I wanted the product to be a zero matrix, but one can also use matrices such as:

Answer: $A = \begin{bmatrix} 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

6.) Circle T for true and F for False (watch out for trick(s)).

[3] 6a.) If A, B, C are matrices and $AB = AC$, then $B = C$ F

For example, take A, B from problem 5 and let C be a zero matrix

[3] 6b.) If A, B, C are square matrices and $AB = AC$, then $B = C$ F

For example, take square matrices A, B from problem 5 and let C be a square zero matrix

[4] 6c.) If A and B are matrices, then $AB = BA$ F

[4] 6.) If A is a 3×3 matrix and the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ had a unique solution, then A is invertible. T

Circle the correct answer:

[5] 7a.) Suppose $A\mathbf{x} = \mathbf{0}$ has a unique solution, then given a vector $\mathbf{b}$ of the appropriate dimension, $A\mathbf{x} = \mathbf{b}$ has

D. at most one solution

[5] 7b.) Suppose A is a SQUARE matrix and $A\mathbf{x} = \mathbf{0}$ has a unique solution, then given a vector $\mathbf{b}$ of the appropriate dimension, $A\mathbf{x} = \mathbf{b}$ has

B. Unique solution