Ch 5 Review Questions:

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix} \overrightarrow{R_2 - R_1 \to R_2, R_3 - 2R_1 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = D$$

0.) Does  $C\mathbf{x} = \mathbf{b}$  have at most one solution for all  $\mathbf{b}$ ?

- 1.) Does  $C\mathbf{x} = \mathbf{0}$  have exactly one solution?
- 2.) In an echelon form of C, is there a leading entry in every COLUMN?
- 3.) Is **0** the only solution to  $C\mathbf{x} = \mathbf{0}$ ?
- 4.) Are the columns of C linearly independent?

5.) Are none of the columns of C a linear comb'n of the other columns of C?

6.) Are none of the columns of C in the span of the other columns of C?

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix} \overrightarrow{R_2 - R_1 \to R_2, R_3 - 2R_1 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = D$$

- 0.) Does  $C\mathbf{x} = \mathbf{b}$  have more than one solution for some  $\mathbf{b}$ ?
- 1.) Does  $C\mathbf{x} = \mathbf{0}$  have an infinite number of solutions?
- 2.) Are there free variables in the solution to  $C\mathbf{x} = \mathbf{0}$ ?
- 3.) Does  $C\mathbf{x} = \mathbf{0}$  have a non-zero solution?
- 4.) Are the columns of C linearly dependent?
- 5.) Is one of the columns of C a linear comb'n of the other columns of C?
- 6.) Is one of the columns of C in the span of the other columns of C?

If possible, write one of the columns of C as a linear combination of the other columns of C:

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix} \overrightarrow{R_2 - R_1 \to R_2, R_3 - 2R_1 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = D$$

- 1.) Does  $C\mathbf{x} = \mathbf{b}$  have at least one solution for all  $\mathbf{b}$ ?
- 2.) Does  $C\mathbf{x} = \mathbf{b}$  have a solution for all  $\mathbf{b}$ ?
- 3.) In an echelon form of C, are there NO rows of all zeros?
- 4.) In an echelon form of C, is there a leading entry in every ROW?

5.) Can any vector in  $R^{-}$  be written as a linear comb'n of the columns of C?

6.) Do the columns of C span  $R^{-?}$ 

1b.) Find a solution to the equation  $C\mathbf{x} = \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$ .

2b.) Write 
$$\begin{bmatrix} 3\\7\\6 \end{bmatrix}$$
 as a linear combination of the columns of  $C$ .

3b.) Write  $3 + 7t + 6t^2$  as a linear combination of  $\{1 + t + 2t^2, 2 + 4t + 4t^2, 3 + 5t + 6t^2, 4 + 4t + 4t^3\}.$ 

1a.) Does 
$$C\mathbf{x} = \begin{bmatrix} 4\\2\\0 \end{bmatrix}$$
 have at least one solution?  
1b.) Does  $C\mathbf{x} = \begin{bmatrix} 3\\7\\6 \end{bmatrix}$  have at least one solution?

2a.) Is 
$$\begin{bmatrix} 4\\2\\0 \end{bmatrix}$$
 a linear combination of the columns of  $C$ :

2b.) Is 
$$\begin{bmatrix} 3\\7\\6 \end{bmatrix}$$
 a linear combination of the columns of  $C$ :

3a.) Is 
$$4 + 2t$$
 a linear combination of  $\{1 + t + 2t^2, 2 + 4t + 4t^2, 3 + 5t + 6t^2, 4 + 4t + 4t^3\}$ ?

3b.) Is 
$$3 + 7t + 6t^2$$
 a linear combination of  $\{1 + t + 2t^2, 2 + 4t + 4t^2, 3 + 5t + 6t^2, 4 + 4t + 4t^3\}$ ?

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix} \overrightarrow{R_2 - R_1 \to R_2, R_3 - 2R_1 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = D$$