Ch 5 Review Questions:
$C=\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8\end{array}\right] \stackrel{R_{2}-R_{1} \rightarrow R_{2}, R_{3}-2 R_{1} \rightarrow R_{3}}{\longrightarrow}\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]=D$
0.) Does $C \mathbf{x}=\mathbf{b}$ have at most one solution for all $\mathbf{b}$ ?
1.) Does $C \mathbf{x}=\mathbf{0}$ have exactly one solution?
2.) In an echelon form of $C$, is there a leading entry in every COLUMN?
3.) Is $\mathbf{0}$ the only solution to $C \mathbf{x}=\mathbf{0}$ ?
4.) Are the columns of $C$ linearly independent?
5.) Are none of the columns of $C$ a linear comb'n of the other columns of $C$ ?
6.) Are none of the columns of $C$ in the span of the other columns of $C$ ?
$C=\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8\end{array}\right] \stackrel{R_{2}-R_{1} \rightarrow R_{2}, R_{3}-2 R_{1} \rightarrow R_{3}}{\overrightarrow{1}}\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]=D$
0.) Does $C \mathbf{x}=\mathbf{b}$ have more than one solution for some $\mathbf{b}$ ?
1.) Does $C \mathbf{x}=\mathbf{0}$ have an infinite number of solutions?
2.) Are there free variables in the solution to $C \mathbf{x}=\mathbf{0}$ ?
3.) Does $C \mathbf{x}=\mathbf{0}$ have a non-zero solution?
4.) Are the columns of $C$ linearly dependent?
5.) Is one of the columns of $C$ a linear comb'n of the other columns of $C$ ?
6.) Is one of the columns of $C$ in the span of the other columns of $C$ ?

If possible, write one of the columns of $C$ as a linear combination of the other columns of $C$ :
$C=\left[\begin{array}{llll}1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8\end{array}\right] \xrightarrow[R_{2}-R_{1} \rightarrow R_{2}, R_{3}-2 R_{1} \rightarrow R_{3}]{\overrightarrow{1}}\left[\begin{array}{cccc}2 & 3 & 4 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]=D$
1.) Does $C \mathbf{x}=\mathbf{b}$ have at least one solution for all $\mathbf{b}$ ?
2.) Does $C \mathbf{x}=\mathbf{b}$ have a solution for all $\mathbf{b}$ ?
3.) In an echelon form of $C$, are there NO rows of all zeros?
4.) In an echelon form of $C$, is there a leading entry in every ROW?
5.) Can any vector in $R$ - be written as a linear comb'n of the columns of $C$ ?
6.) Do the columns of $C$ span $R$ ?

1b.) Find a solution to the equation $C \mathbf{x}=\left[\begin{array}{l}3 \\ 7 \\ 6\end{array}\right]$.

2b.) Write $\left[\begin{array}{l}3 \\ 7 \\ 6\end{array}\right]$ as a linear combination of the columns of $C$.

3b.) Write $3+7 t+6 t^{2}$ as a linear combination of $\left\{1+t+2 t^{2}, 2+4 t+4 t^{2}, 3+5 t+6 t^{2}, 4+4 t+4 t^{3}\right\}$.

1a.) Does $C \mathbf{x}=\left[\begin{array}{l}4 \\ 2 \\ 0\end{array}\right]$ have at least one solution?

1b.) Does $C \mathbf{x}=\left[\begin{array}{l}3 \\ 7 \\ 6\end{array}\right]$ have at least one solution?

2a.) Is $\left[\begin{array}{l}4 \\ 2 \\ 0\end{array}\right]$ a linear combination of the columns of $C$ :

2b.) Is $\left[\begin{array}{l}3 \\ 7 \\ 6\end{array}\right]$ a linear combination of the columns of $C$ :

3a.) Is $4+2 t$ a linear combination of
$\left\{1+t+2 t^{2}, 2+4 t+4 t^{2}, 3+5 t+6 t^{2}, 4+4 t+4 t^{3}\right\} ?$

3 b .) Is $3+7 t+6 t^{2}$ a linear combination of
$\left\{1+t+2 t^{2}, 2+4 t+4 t^{2}, 3+5 t+6 t^{2}, 4+4 t+4 t^{3}\right\}$ ?
$C=\left[\begin{array}{llll}1 & 2 & 3 & 4 \\ 1 & 4 & 5 & 4 \\ 2 & 4 & 6 & 8\end{array}\right] \xrightarrow[R_{2}-R_{1} \rightarrow R_{2}, R_{3}-2 R_{1} \rightarrow R_{3}]{\overrightarrow{1}}\left[\begin{array}{ccc}1 & 3 & 4 \\ 0 & 2 & 2\end{array} 0010.0 ~ D\right.$

