22M174/22C174: Optimization techniques.

Homework 2. Due 02/13/13.

1. To find the zero x^* of $g(x) := \sqrt{x}e^x - 1 = 0$ in the interval $[0.1, 1]$ apply 10 iterations of

   (a) the bisection method starting with $a := 0.1, b := 1$. How many iterations k of the bisection method ensure an absolute error of 10^{-7} on x^*, i.e., $|c_k - x^*| < 10^{-7}$?

   (b) the simplified Newton iterates starting with $x_0 := 0.55$;

   (c) the Newton iterates starting with $x_0 := 0.55$;

   (d) the secant iterates starting with $x_0 := 0.55, x_1 := x_{1,\text{Newton}}$;

   (e) the fixed-point iterates $x_{k+1} = e^{-2x_k}$ starting with $x_0 := 0.55$.

   For each method compute also the absolute errors $|x_k - x^*|$ for each iterate. Note: the zero is $x^* = 0.426302751006863\ldots$.

2. Let $g : \mathbb{R} \to \mathbb{R}$ be twice continuously differentiable and $x^* \in \mathbb{R}$ a zero satisfying $g(x^*) = 0$ and $g'(x^*) \neq 0$. Show that in a neighborhood of x^* Newton's method is a contraction mapping.

3. To find a zero of $g(x) := 4x^5 - x^3 - x$ apply 10 iterations of Newton's method starting with $x_0 := 0.5$. Print x_k for $k = 0, \ldots, 10$. Do these iterations converge?

4. To find a zero of $g(x) := x^6 - (3/2)^6$ apply 6 iterations of the secant method starting with $x_0 := 0.25$ and $x_1 := 5$. Print x_k for $k = 0, \ldots, 6, 7$. Do these iterations seem to converge numerically?

5. To find an approximation to the minimimizer of the function

   $$f(x) = 2x^{3/2} + 3e^{-x} + 5$$

   on the interval $[a_0, b_0] := [0.2, 0.6]$ apply 3 steps of the golden section method, i.e., give the 3 intervals of uncertainty $[a_1, b_1]$, $[a_2, b_2]$, and $[a_3, b_3]$ obtained with that method.

6. In the golden section method prove that we obtain $\alpha_{k+1} = \beta_k$ when $f(\alpha_k) > f(\beta_k)$.