22M174/22C174: Optimization techniques.

Homework 8. Due 04/19/13.

1. Find the points in the set

$$X := \{x \in \mathbb{R}^3 \mid x_1 x_3 - x_2^2 + 4 = 0\}$$

   which are the closest to the origin $[0,0,0]^T$ (closest in the sense of the Euclidean distance).

2. We consider minimizing the quadratic function

$$q(x_1, x_2) = x_1^2 + x_2^2 - 3x_1x_2 + x_1 + x_2$$

   under the constraint $2x_1 + x_2 = 2$. A basis Z of the null-space of the Jacobian of $c_1(x_1, x_2) := 2x_1 + x_2 - 2$ is given by one the following vectors

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ -2 \end{bmatrix}.$$

   Which one? Find the stationary points $(x^\star, \lambda^\star)$ of the Lagrangian. Then evaluate $Z^T \nabla q(x^\star)$ and $Z^T \nabla^2 q(x^\star) Z$. Give the global minimizer(s) of $q(x_1, x_2)$ in the set

$$X = \{x \in \mathbb{R}^2 \mid 2x_1 + x_2 = 2\}.$$

3. Prove the lemma on p. III.16 of the class notes.

4. Find the global minimizer(s) of $f(x_1, x_2) = (x_2 - 2)^2 + 2(x_1 - 1)^2$ in the set

$$X := \{x \in \mathbb{R}^2 \mid x_1 \geq x_2^2,\ x_1 + x_2 \leq 2\}.$$