22M:041, Secs. 091 and 121: Differential Equations for Engineers

Background on the use of MAPLE and of the UNIX machines in B5 MLH

9/18/01

This is not an assignment! The first few paragraphs immediately below are meant to help you get comfortable with MAPLE, and (unless you want to use other equipment) the workstations in B5 MLH.

When you get a feel for MAPLE and what it can do, it will become clear that MAPLE is a great tool for projects in many different subjects – not just Mathematics. As a program for symbolic calculation, it is rivalled only by *Mathematica*. It produces beautiful plots and graphics to illustrate its results; or the graphics *are* the results, in cases where things cannot be written in closed form. (As an extreme example, think of weather prediction.)

Remember to get an early start on each computer assignment, and try to get questions (about things that aren't working for you) out in the open soon. Otherwise ... things happen, and whenever computers and printers are involved, there are glitches. You can always e-mail me (branson@math.uiowa.edu) with questions. Usually it's good to send a copy of the MAPLE session that's giving you trouble. If you don't know how to send the MAPLE session along, let me know this in your e-mail note; I'll give you instructions.

You're actually free to use any symbol cruncher (MAPLE, *Mathematica*, Matlab, ...) you wish, as long as it's capable of getting the job done. Similarly, you can work on your favorite platform (UNIX machines in B5 MLH, UNIX machines in the Engineering Building, PC, or Mac). However, detailed *support* will only be offered on MAPLE, and for the most part only on UNIX stations. A note to dedicated Mac users who use UNIX stations for the first time: no spaces are allowed in UNIX filenames! Or, put another way, no multi-word filenames.

Getting started in B5 MLH, and in MAPLE. You should be able to enter room B5 MLH by swiping your ID card through the card reader at the door. Your account (assigned in class) should be good on any of the machines there; these are HP workstations operating under UNIX. You can send and recieve e-mail from this account; your address is userid@divms.uiowa.edu. If you want to log in remotely from somewhere else, you need to choose one of the machine names – for example you can telnet or rlogin to garlic.divms.uiowa.edu or cinnamon.divms.uiowa.edu, but **not** just to divms.uiowa.edu.

Once you've logged in, you can open a terminal window by clicking on an icon near the bottom of your screen; this icon looks like a picture of a monitor. Notice that your visual environment has different *workspaces*, accessible by clicking on the icons One, Two, Three, etc. at the bottom of the screen. You can launch windows and/or other jobs (one or many) in each workspace. When you want to log out, you do so by holding down the leftmost mouse button, with the cursor outside all windows. You'll get a "Workspace Menu", and there you drag the cursor down to "Log out...".

Of course you don't need a command line to launch MAPLE – you can start it from the menu that you get by holding down the rightmost mouse button. (You can also get Matlab and *Mathematica* from the same menu.) If you want to launch MAPLE from the command line, type maple or xmaple at the prompt. maple will open a command line version of MAPLE, in which, for example, very rough versions of graphs will be drawn using typewriter symbols. It is really too primitive for our purposes, especially when we want to produce graphs (or *plots*). xmaple opens the Xwindows version of MAPLE (the same version you'd get by launching from the menu); here the type fonts and plots look very nice. You can get out of either version of MAPLE by typing "quit" at the > prompt. You can also get out of xmaple by choosing "Exit" from the file menu. All of what follows assumes you are working in an xmaple session, or the equivalent on a PC or Mac.

You can, of course, use Netscape from your account. Netscape can be called from the same menu as MAPLE, or from the command line. To get to the course page, go to my home page, http://www.math.uiowa.edu/~branson, and follow the link there (not too far from the top of the page).

Note the semicolon (;) at the end of the command. MAPLE requires either a semicolon or a colon to signal that the instruction should be compiled. If you just hit *Return*, you will go to the next line, but MAPLE will think the command from the previous line is just continuing on the new line. Improperly closed instructions can ruin later instructions, even when the later ones are entered correctly. If a command is ended with a semicolon, MAPLE gives a response; usually repeating the command, or giving the result of some task the command has asked it to do. If a command is ended with a colon (:), MAPLE compiles the command, but keeps silent. A semicolon is almost always better: you can find out how MAPLE interpreted the command you just gave, and whether it is thinking what you think it is thinking.

Several options in the File Menu of an xmaple session will be important for us. Most important is "Save As" – this allows you to save your MAPLE worksheet to a file, whose name you choose, in order to reload it later and work on it further. The guiding principle is: Try never to retype what you have typed correctly once. You should save the worksheet as a file with extension **.mws** – this makes it a file readable by xmaple. The dialog box gives you the flexibility to store the file anywhere in your directory structure. You can reload your worksheet for further work by launching xmaple, then choosing "Open" from the File Menu. You then choose which file to open, and again the whole directory structure of your account is available. There is also a list of the last few **.mws**-files you've worked on; it's usually easiest just to click on one of these.

Another option which can be useful (and which doesn't preclude your also using the other options described above, especially "Save As") is "Export As". If you export your session as a plain text file, you get a list of your commands that is e-mailable and readable by someone who might not have access to MAPLE at the moment he or she reads it. This is useful when you need to ask for help on things that "should work" but just don't – the hope is that the other person can see the source of the glitch by reading your raw commands. You can also export a session as an HTML file – any graphs in the session will be made into separate .gif-files that will be called by the main HTML file. Thus you can post your session on the Web in such a way that anyone can read it (even those without the capability to read .ps- or .pdf-files).

Note that you can move the cursor within a MAPLE worksheet and correct or change previous lines. There are also options under other menus that allow you to add new MAPLE prompt lines, or delete old lines or groups of lines. You can also change the format of your worksheet in various ways;

please experiment.

One important thing to keep in mind is that **your worksheet's logical state is not necessarily the same as its apparent state**. The lines of your session are viewed by MAPLE as being in the order in which they were **executed**, rather than the order in which they **appear** on the page. For example, if you change something on line 2 that has consequences on line 17, you can only make this felt on line 17 if you step through (using Return) all relevant intervening instructions. Likewise, you will have to step through your already-typed instructions after reloading a worksheet to attain the proper logical state. You can also choose "Execute worksheet" from the Edit menu to cause MAPLE to step through all instructions.

It's advisable to experiment a little with the above tips and principles with **small** worksheets to make sure you have a feel for how things work – then, by the time you're working with a fairly elaborate worksheet, you'll be confident that all manipulations work for you, and that your session won't end up in a field somewhere in Wyoming.

To print a worksheet, choose "Print" from the File Menu. You'll get a choice of "printing" in the sense of making a .ps-file, or printing to a printing device; usually, you want to take the latter. The result should come out on B5's printer, whose name is **pb5**. You may need to supply your own paper in the printer; there is no supply kept in room B5.

I would like you to hand in computer assignments by printing your worksheet (after you've cleaned it up and gotten it to a near-perfect state), and handing in the paper copy. Occasionally, there are mysterious problems with printing (like fonts needed for a plot that are unavailable); if this happens, you may turn in your assignment by e-mailing your xmaple session to me.

Integration using MAPLE. As we know from hard experience, our ability to find closed form solutions to differential equations is heavily dependent on our ability to integrate. In fact, all closed-form solution methods are really strategies aimed at reducing a DE problem to one or more integration problems. What is true for us is also true for automated symbol crunchers like MAPLE.

The integration command in MAPLE is "int". For example, to get $\int \cos(t)dt$, type

$$\mathsf{int(cos(t), t)}$$

MAPLE returns the output $\sin(t)$. Note that the constant of integration is omitted – MAPLE gives you *one* antiderivative of the integrand. Actually, you *can* see the constant of integration explicitly, if you want, by putting the integration problem to MAPLE *as a differential equation*:

$$\mathsf{dsolve(diff(f(t), t) = cos(t), f(t));}$$

To get a definite integral like $\int_0^{\pi/2} \cos(t)dt$, type

$$\mathsf{int(cos(t), t = 0..Pi/2);}$$

The ".." construction (as in t=0..Pi/2) occurs throughout MAPLE, to indicate that a variable runs through certain values. Depending on the command, the variable may run continuously, or discretely in evenly spaced steps. In the case of int, the variable should run continuously, of course. Pi is MAPLE's name for π.

Some integration problems will induce MAPLE, in its reply, to use functions you've never heard of. For example, ask MAPLE to compute $\int xe^x(\ln x)dx$:

$$\mathsf{int(x * exp(x) * ln(x), x);}$$

The function "Ei" will appear. To find out what it is, ask MAPLE:

?Ei

You don't need a semicolon or colon after one of these "help" requests. In fact, you can find out about any MAPLE command or mathematical procedure this way. For example, there is a very useful command called isolate – to see what it does, type

?isolate

The help pages are arranged with hierarchical content, which means that (as on the Web) you can click on certain words, and get more information on the associated topics.

MAPLE no longer has a name reserved for e, the base of natural logarithms. (Formerly, E was the reserved name of this.) Now, e and E will just be interpreted by MAPLE as variable names. The exponential function e^x is exp(x). The number e may be obtained as exp(1).

Note that the * must always be included whenever multiplication is intended; a*b is the product of a and b, while ab is a single variable name. ln is MAPLE's name for the natural logarithm.

In the problems below, see whether MAPLE can do the integrations. In the case of an indefinite integral, check the answer (if any) by differentiating. In the case of a definite integral with no free variables, if MAPLE cannot come up with an answer (that is, just returns the same integration problem), try to get a number by using evalf to request a *full evaluation*. As examples of checking by differentiating and full evaluation, consider the following. We would like to compute

$$\int \frac{x}{x^3+1} dx.$$

We type

```
int(x/(x^3+1), x);
```

If all is well, we get an answer, a function that is somewhat more complicated than the integrand was. To check this answer, we type

```
diff(%, x);
```

The percent sign % is MAPLE's notation for "the last answer"; this little device sometimes saves us some secretarial work. The abbreviation for "the second last answer" is %%, and so on.

What we just got upon differentiating should be the original integrand, $x/(x^3+1)$, but it (probably) looks different. MAPLE has a number of procedures which rewrite expressions in forms we desire; some of these will be encountered below. Something that's often worth a try is

```
simplify(%);
```

In the present case, what does the job is

```
normal(%, expanded);
```

To see why we chose this, and what it might be expected to do in other cases, just put the question to MAPLE with

```
?normal
```

As an example of a full evaluation, try to compute

$$\int_0^\infty e^{-x} x^{3/2} \ln(x) \cos(x^2) dx.$$

We make the question understood to MAPLE by typing

$$\mathsf{int(exp(-x) * x^{(3/2)} * ln(x) * cos(x)^2), x = 0..infinity);}$$

MAPLE will (probably) just return the problem as its answer – it's not able to come up with a number (which we might expect to be written symbolically in terms of π and e). However, as the next resort, we can ask MAPLE to perform the integration numerically:

$$\mathsf{evalf}(\%);$$

This should produce an answer (after some time).

Practice Problems. In each problem, have MAPLE do the integral. Then, in the case of indefinite integrals ($\int$ as opposed to $\int_a^b$), differentiate the answer to see whether it agrees with the original integrand. (Often some simplification is required to get things to agree.)

1. $\int_0^\pi \sin^2(t) dt$
2. $\int x e^x (\ln x) dx$
3. $\int (\ln t) dt$
4. $\int x^7 (\ln x) dx$
5. $\int e^t \cos(t) dt$
6. $\int e^{-x^2} dx$
7. $\int_{-\infty}^\infty e^{-x^2} dx$
8. $\displaystyle\int \frac{t}{t^5 + 1} dt$
9. $\displaystyle\int_0^\infty \frac{t^4 (\ln t)^2}{(1 + 3t^2)^3} dt$

10. $\int_0^\infty t^7 e^{-t} dt$

11. $\int x e^{-x^2} dx$

12. $\int_0^t e^u \cos(t-u) du$

13. $\displaystyle\int \frac{y^2+y+1}{y^3-y} dy$

14. $\int_0^\infty t^a e^{-bt} dt \quad (a, b \text{ constants})$

15. $\displaystyle\int \frac{1}{1-v^{3/2}} dv$

Hint: In the last integral, when you check your answer by differentiating, try ?radsimp to simplify what you get.

Direction fields. We can get a first glimpse of MAPLE's graphical power by plotting some *direction fields*. Recall that for the first order differential equation

$$\frac{dy}{dt} = F(t, y),$$

the direction field is a ty-plane with an "iron filing" of slope $F(t, y)$ placed at each point (t, y). Solution curves have to follow the iron filings.

Here's an example of a direction field instruction, designed to draw a direction field for the differential equation $y' = y(y-1)(y-2)$:

```
dfieldplot([diff(x(t), t) = 1, diff(y(t), t) = y * (y - 1) * (y - 2)],
    [x(t), y(t)], t = -2..2, x = -1..2, y = -1..2);
```

The dfieldplot instruction works on *autonomous* pairs of equations $x'(t) = F(x, y)$, $y'(t) = G(x, y)$. (*Autonomous* means that the independent variable t is missing from the right-hand sides.) To get the direction field of an equation $y' = H(t, y)$, we "artificially" make it into such a pair $x'(t) = 1$, $y'(t) = H(x, y)$. For more information about the dfieldplot command, type

?dfieldplot

This will show you how to get arrows (iron filings) of different thicknesses, colors, etc. It's often a good idea to pull the example instructions from a help page into your worksheet, then tweak the parameters a little to produce your own pictures, taking advantage of all the formatting work that the MAPLE people have already put into the examples. This is a good way to learn to use any single-instruction routine – imitation and variation of the canned examples.

A large part of the skill and art of plotting with a program such as MAPLE is getting an informative picture. For example, if you plot too large a part of the xy-plane, and all the interesting behavior is near the origin, then the phenomenon you seek to picture may be reduced to a low-resolution speck in a sea of vertical or horizontal iron filings. You could also have the opposite problem, focussing initially on too small a region in the xy-plane, and missing a "tornado" that might live just to the left or right of your picture. In this case, you may have to expand the picture, or "move over" to capture the interesting behavior, and ultimately make the perfect plot.

Practice Problems. In the following problems, let a be the $7^{\underline{\text{th}}}$ digit in your University number, and let b be the $9^{\underline{\text{th}}}$ digit. In each case, if the digit is 0, use 10 instead.

Plot direction fields for the following equations:

1. $\dfrac{dy}{dt} = y(y-a)(y-b)$.
2. $\dfrac{dy}{dt} = -\dfrac{bt+ay}{y}$.
3. $\dfrac{dy}{dt} = ay(b-y)$.
4. $\dfrac{dy}{dt} = -ay(b-y)$.