Review for Final Exam of Math 033

The FINAL will be hold on Monday May 12, 2:15pm.

The key words for the exam are basic geometry of R^3:lines, planes, rotations, basis, Least Square, linear systems. The exam will be comprehensive.

(1) We will possibly retest things about basis, determinants.

(2) Orthonormal basis.

(3) Least Square and the best line fits.

(4) Orthogonal projections, section 5.3.

(5) Distance to the plane.

1. Find the following determinant and its geometric meaning.

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

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2. Find the projection of $(1, 2)$ to the line $x + 3y = 0$.
3. Find the project of e_1, e_2 to the plane $x + 2y + 3z = 0$.
4. Find all the solutions of $\begin{cases} x + y + 2z + w = 6 \\ 2x - 6z = 1 \end{cases}$.
5. Find dimension and basis of $x + y - 2z = 0$.
6. Find an orthonormal basis of the plane $x + y - 2z = 0$.
7. Find the matrix of rotations of R^2 by a angle of 30^o clockwise. Rotate $(1, 2)$ clockwise 30^o.
8. Find the distance of $[1 \ \ 2 \ \ 3]$ to the plane $x + 2y + 3z = 1$.
9. Find the project of e_1, e_2, e_3 to the plane $x + y + z = 0$.
10. Find bases and dimensions of row, column and null spaces of the following subspaces.

$$\begin{bmatrix} 1 & -2 & 1 & 1 \\ 2 & -4 & 0 & 0 \end{bmatrix}.$$

11. Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

and orthogonally diagonalize it.

12. Analyze (that is, find axes and classify) the quadric surface represented by

$$x^2 - 2xy + 4y^2 + 4z^2 = 6.$$

13. Find best line fit of the points: $(0, 1), (1, 3), (2, 2)$.

14. Find the dimensions and bases of the volume, raw and null space of the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}.$$

15. Find $[1 \ \ 2 \ \ 3] \times [1 \ \ 1 \ \ 1]$.

16. Find the eigenvalues and eigenvectors for

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}.$$

17. Find

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & -1 \\ 2 & 1 \end{bmatrix}.$$

18. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

19. Write $(1, 2, 1)$ as a linear combination of $(1, 1, 1), (0, 1, 1)$ and $(0, 0, 1)$.

20. The distance of $(1, 2)$ to the line $x + y = 5$.