The second midterm will cover from 13.8 to 15.3.
Here are hints to the problems.

1.Find maxima and maximal value of the function $2x - y$ inside the unit circle.
Let $f(x,y) = 2x - y$. Then we see $f_x = 2 \neq 0$ so there is not critical point for f and hence f couldn't have interior extrema.
Now we look at the boundary $g(x,y) = x^2 + y^2 = 1$.
We have

$$\begin{cases} 2 &= \lambda 2x \\ -1 &= \lambda 2y \\ 1 &= x^2 + y^2 \end{cases}$$

, which leads to $x = \frac{1}{\lambda}, y = -\frac{1}{2\lambda}$, and hence

$1 = \frac{1}{\lambda^2} + \frac{1}{4\lambda^2}$. Final $\lambda = \pm\frac{\sqrt{5}}{2}$, from which we have two points
$\pm\left(2/\sqrt{5}, -1/\sqrt{5}\right)$. A direct computation implies, max is at ... and min is at ....

2. Find the integral $\int\int\int_B x^2 dvol$, where B is the unit ball
$x^2 + y^2 + z^2 \leq 1$.
We use the symmetry of the ball to see that $\int\int\int_B x^2 dvol = \int\int\int_B y^2 dvol = \int\int\int_B z^2 dvol$
and
$\int\int\int_B x^2 dvol = \frac{1}{3}\int\int\int_B (x^2 + y^2 + z^2) dvol = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho^2\rho^2 \sin\phi d\rho d\phi d\theta = \frac{4\pi}{15}$.

3. Find maxima and maximal value of the function $x^2 + 2y$ in the half disk

$x^2 + 3y^2 \leq 1$ and $x \geq 0$.

Similar to 1. Set $f(x,y) = x^2 + 2y$, then $f_x = 2x$ and $f_y = 2$, which implies no interior critical points. On the boundary, set $g(x,y) = x^2 + 3y^2 = 1$,
we have

$$\left\{\begin{array}{ll} 2x &= \lambda 2x \\ 2 &= \lambda 6y \\ x^2 + 3y^2 &= 1 \end{array}\right\}$$

, which leads to solutions $(0, \pm 1/\sqrt{3}), (\pm\sqrt{2/3}, 1/3)$.
A direct comparison will yield the result.

4. Find the integral $\int\int\int_D x dvol$, where D is the part of the unit ball
$x^2 + y^2 + z^2 \leq 1$ and $x \geq 0, y \geq 0, z \geq 0$.
We use spherical coordinates.
$\int\int\int_D x dvol = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 \rho \sin\phi \cos\theta \rho^2 \sin\phi d\rho d\phi d\theta = \frac{1}{3}\frac{\pi}{4} = \frac{\pi}{12}$.

5. Find the extrema of the function $x^2 y$.in the triangle bounded by the $x - axis$

and $y - axis$ and the line $x + y = 1$.

Let $f(x,y) = x^2 y$, then $f_x = 2xy, f_y = x^2$.

For interior critical points, we compute:
$2xy = x^2 = 0$, which leads to $x = 0$, namely all points on the y-axis are critical points.
However, these points are not in the interior.
The triangle has three sides and one the axes, the function is 0 so we don't have
to do calculus there. On the side that $x + y = 1$, we proceed as

$$\left\{ \begin{array}{ccc} 2xy & & = \lambda \\ x^2 & & = \lambda \\ x & +y & = 1 \end{array} \right\}$$

and therefore we have the point $(2/3, 1/3)$. Finally we have to compare
the value of f at $(2/3, 1/3)$ with the value on the axes, which is 0.
This implies that the axes are the minimal with minimal value 0 and
$(2/3, 1/3)$ is the maximum with maximal value .....

6. Compute $\int\int_D x^2 y dx dy$, where D is the upper half disk.
We compute
$\int\int_D x^2 y dx dy = \int_0^\pi \int_0^1 r^4 \cos^2\theta \sin\theta dr d\theta = -\frac{1}{15}\cos^3\theta|_0^\pi = 2/15.$

7. Compute the divergence and curl of the vector fields x^2yi+$\cos(x+y)$j+zk. The physical meaning of divergence is the density of the source of the vector field and that for curl is the vector valued circulation density.
Easy.

8.Compute $\int_C yxdx + xdy$, where C is the counterclockwise unit circle.
Let $x = \cos t, y = \sin t$. Then

$$\int_C yxdx + xdy = \int_0^\pi \cos\theta \sin\theta d\cos\theta + \cos\theta d\sin\theta =$$
$$= \frac{\pi}{2}.$$

9. Compute $\int_S x^2 ds$, where S is the upper half circle.
As in 8,
$\int_S x^2 ds = \int_0^\pi \cos^2\theta d\theta = \frac{\pi}{2}.$
10. Compute the center of mass of unit disk with density $d(x,y) = x^2$.
Clearly $(0,0)$.

11. Write down the change of variable formulae for

spherical and cylindrical coordinates.
In the book.

12. Compute the integral
$\int\int_{x^2+4y^2\le 4}(x+y^2)dxdy.$
Here you better substitute $2y$ to y so the domain of intergration is

the unit disk, this is the change of variable formula:
$\int\int_{x^2+4y^2\le 4}(x+y^2)dxdy = \int\int_{x^2+y^2\le 4}(x+y^2/4)dxdy/2 = \int\int_{x^2+y^2\le 4} y^2/8dxdy$
here we have used $\int\int_{x^2+y^2\le 4} xdxdy = 0$ from the symmetry.
Hence

$$\int\int_{x^2+4y^2\le 4}(x+y^2)dxdy = \frac{1}{16}\int\int_{x^2+y^2\le 4}(x^2+y^2)dxdy = \frac{1}{16}\int_0^{2\pi}\int_0^2 r^3 drd\theta = \frac{\pi 2^5}{2^6} = \frac{\pi}{2}.$$