$x_1 \quad y_1 \quad x_2 \quad y_2$

1. a. Find the distance between the points $(-3,2)$ and $(5,8)$.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(5-(-3))^2 + (8-2)^2}$$

$$= \sqrt{(+8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$$

b. What is the midpoint of the line segment joining the points $(-3,2)$ and $(5,8)$.

$$\left(\frac{x_1+x_2}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{-3+5}{2}, \frac{2+8}{2}\right) = (1,5)$$

2. Find an equation of the form $y = mx + b$ (slope-intercept form) for the line containing the points $(1,3)$ and $(-1,-2)$.

$$\text{Slope } m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-3}{-1-1} = \frac{-5}{-2} = \frac{5}{2}.$$

$y = mx + b$

$y = \frac{5}{2}x + b$

Since $(1,3)$ is on the graph.

y

$x=1$

$3 = \frac{5}{2} + b$

$b = \frac{1}{2}$

$$\boxed{y = \frac{5}{2}x + \frac{1}{2}}$$

3. Find the center (h, k) and radius r of the circle $x^2 + y^2 - 2x - 4y - 4 = 0$. Graph the circle.

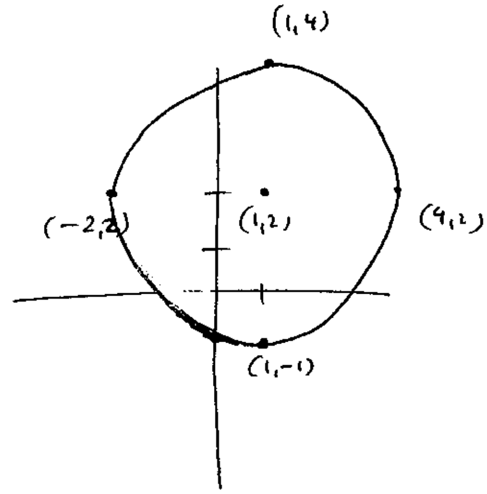
$$\left(x^2-2x+1\right)-1+\left(y^2-4y+4\right)-4-4=0$$

half square half/square 2

$$(x-1)^2+(y-2)^2-9=0$$

$$(x-1)^2+(y-2)^2=9=3^2$$

Center $(1,2)$, radius 3



4. Let $f(x) = 1 + \frac{1}{x}$ and $g(x) = \frac{1}{x}$ Find the domains of f, g, $f - g$ and $\frac{g}{f}$.
You MUST indicate which is which.

$\frac{1}{x}$ is not calculable if $x=0$.

Domain of f : All reals except 0 or $(-\infty,0)\cup(0,\infty)$

Domain of g : " " " "

Domain of $f-g$ " " " "

(Since $f(0)$, $g(0)$ are not defined, $(f-g)(0)$ is not defined)

(Although, $f(x)-g(x)=1$ for all $x\neq 0$)

Domain of $\frac{g}{f}=\frac{\frac{1}{x}}{1+\frac{1}{x}}$ is all reals except 0 and -1.

0 is excluded since $f(0)$, $g(0)$ are not defined.

-1 is excluded since $f(-1)=1+\frac{1}{-1}=0$, and we can't divide by 0.

5. Is the function $g(x) = -3x^2 - 5$ even, odd, neither or both? Algebraically determine your answer.

$g(-x) = -3(-x)^2 - 5 = -3(-x)(-x) - 5 = -3x^2 - 5 = g(x)$

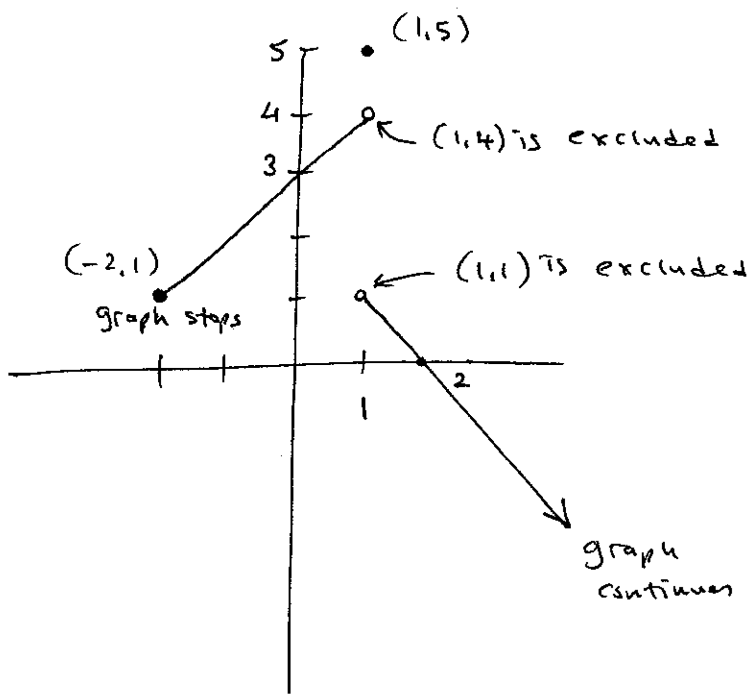
So $g(-x) = g(x)$ and hence $g(x)$ is even: 9 points!

Since there is a function $f(x) \equiv 0$ which is both odd and even, we must show that $g(x)$ is not odd, otherwise the answer would be "both".

$g(x)$ is not odd since $\left.\begin{array}{l} g(1) = -8 \\ g(-1) = -8 \end{array}\right\}$ hence $\underset{-8}{g(-1)} \neq \underset{8}{-g(1)}$, $-8 \neq 8$.

6. Plot the graph of the function $h(x) = \begin{cases} x+3 & \text{if } -2 \le x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$.

List its domain, range and all x–intercept(s) and y–intercept. Identify which is which, and state none if there is not one.

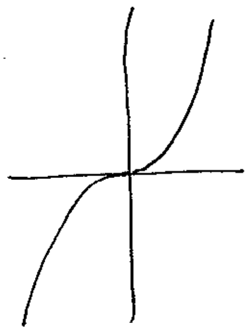


Domain $[-2, \infty)$ or $x \geq -2$

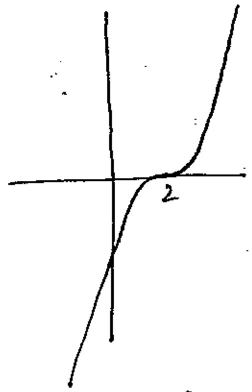
Range $(-\infty, 4) \cup \{5\}$ or ($y < 4$ or $y = 5$)

$h(0) = 3 = y$ y-intercept and $h(2) = 0$, so $x = 2$ only x-int.

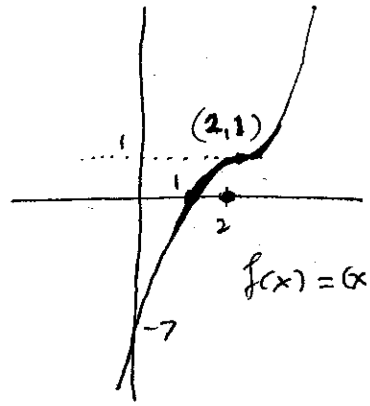
7. Plot the graph of $f(x) = (x-2)^3 + 1$ by using the techniques of shifting, compressing, stretching, or reflecting. Find all intercepts.



$g(x) = x^3$



$h(x) = (x-2)^3$



$f(x) = (x-2)^3 + 1$

y-int.

$f(0) = (-2)^3 + 1 = -8 + 1 = -7$

$f(x) = (x-2)^3 + 1 = 0$

$(x-2)^3 = -1$ x-int

$x - 2 = -1 \rightarrow x = 1$

8. The function $T(x) = 0.15(x - 7300) + 730$ represents the tax bill T of a single person whose adjusted gross income is x dollars for the income between \$7,300 and \$29,700, inclusive in 2005.

a. What is the domain of this function?

$7300 \leq x \leq 29{,}700$, or $[7{,}300, 29{,}700]$

b. What is a single filer's tax bill if adjusted gross income is \$18,000?

$T(18000) = 0.15(18{,}000 - 7{,}300) + 730 = 0.15(10{,}700) + 730$

$= 1605 + 730 = \$2{,}335$

c. What is a single filer's adjusted gross income if tax bill is \$2,860?

Solve

$2860 = T(x) = 0.15(x - 7300) + 730$

$2860 - 730 = 0.15(x - 7300)$

$2130 = 0.15(x - 7300)$

$\dfrac{2130}{0.15} = x - 7300$

$1420 = x - 7300 \implies x = \$21{,}500$

9. a. Graph the quadratic function $h(x) = 2x^2 - 8x$, by determining whether its graph opens up or down, by finding its vertex, axis of symmetry, y-intercept, x-intercepts if any.

b. Determine the domain, the range of the function.

c. Determine where the function is increasing and where it is decreasing.

$a = 2$ $\quad a > 0 \implies$ opens up $\cup$

$b = -8$

$c = 0$

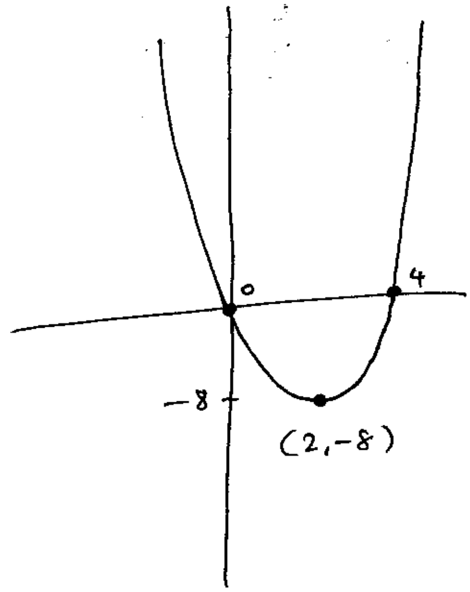
$x = 2$ axis of symmetry

$$\left.\begin{aligned} -\frac{b}{2a} &= \frac{8}{4} = 2 \\ h(2) &= 8 - 16 = -8 \end{aligned}\right\} \text{ vertex } (2,-8)$$

$h(0) = 0$, $\quad y = 0$ y-intercept.

$0 = h(x) = 2x^2 - 8x = 2x(x-4)$

$x = 0, 4$ x-intercepts.



Domain $= \mathbb{R}$

Range $= [-8, \infty)$ or $y \geq -8$

f is increasing on $(2, \infty)$

f is decreasing on $(-\infty, 2)$

10. Let $f(x) = x^2 - 1$ and $g(x) = 3x + 3$.

a. Solve $f(x) = 0$

$x^2 - 1 = 0$

$(x-1)(x+1) = 0$ $\quad x = 1$ or -1.

b. Solve $g(x) < 0$

$3x + 3 < 0 \implies 3x < -3$

$\implies x < -1$.

c. Solve $g(x) = f(x)$

$x^2 - 1 = 3x + 3$ $\quad a = 1$

$x^2 - 3x - 4 = 0$ $\quad b = -3$, $c = -4$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 16}}{2}$$

$$= \frac{3 \pm 5}{2} = 4, -1.$$

d. Solve $g(x) < f(x)$

$3x + 3 < x^2 - 1$

$0 < x^2 - 3x - 4 = (x-4)(x+1)$

x	test -2	-1	test 0	4	test 5
$x^2 - 3x - 4$	$+$	ϕ	$-$	ϕ	$+$
	solution				Solution

Answer

$(-\infty, -1) \cup (4, \infty)$

or

$x < -1$ or $x > 4$.