

**22m9.Evening section**  
**Practice test for the Final Exam**

1. Establish the following identity:

$$\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} = 2 \sec v$$

2. If  $\cos \theta = -\frac{1}{4}$  and  $\pi < \theta < \frac{3\pi}{2}$ , find the exact values of the following:

- $\sin \theta$
- $\cos\left(\theta + \frac{\pi}{3}\right)$
- $\sin 2\theta$
- $\cos \frac{\theta}{2}$

3. a. Find all solutions of  $\sin(3\theta) = -1$  with  $0 \leq \theta < 2\pi$ .

b. Find all solutions of  $\sin^2 \theta - \cos^2 \theta = 1 + \cos \theta$  with  $0 \leq \theta < 2\pi$ .

4. Solve the triangle  $\triangle ABC$ , if  $b = 1, c = 3$  and  $A = \frac{\pi}{6}$ .

5. a. Write the equation  $x^2 + y^2 = y$  using the polar coordinates  $(r, \theta)$ .

b. Write the equation  $r = \sin \theta + \cos \theta$  using the rectangular coordinates  $(x, y)$ .

6. Identify and graph each of the following polar equations.

- $r = 2 \sin \theta$
- $r \sin \theta = 2$

7. Plot the graph of  $f(x) = \tan\left(\frac{\pi}{2}x\right)$ . State its domain, range, period, all vertical asymptotes and all intercepts. Provide 3 points of the graph with both coordinates.

8. Plot the graph of  $f(x) = 2 \cos\left(3x + \frac{\pi}{2}\right)$ . State its domain, range, period, amplitude and phase shift. Provide 3 points of the graph with both coordinates.

9. Find the exact values of the following.

- $\sin^{-1}(-1)$
- $\tan^{-1}(1)$
- $\sin^{-1}\left(\sin \frac{9\pi}{8}\right)$

10. Find the exact value of  $\tan\left[\sin^{-1}\left(-\frac{1}{3}\right)\right]$ .

ANSWERS:

1.

$$\begin{aligned}\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} &= \frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} \cdot \frac{1 + \sin v}{1 + \sin v} \\ &= \frac{1 - \sin v}{\cos v} + \frac{(\cos v)(1 + \sin v)}{1 - \sin^2 v} \\ &= \frac{1 - \sin v}{\cos v} + \frac{(\cos v)(1 + \sin v)}{\cos^2 v} \\ &= \frac{1 - \sin v}{\cos v} + \frac{1 + \sin v}{\cos v} \\ &= \frac{2}{\cos v} = 2 \sec v\end{aligned}$$

2.

a.  $\sin \theta = \frac{-\sqrt{15}}{4}$

b.  $\cos\left(\theta + \frac{\pi}{3}\right) = \frac{3\sqrt{5}-1}{8}$

c.  $\sin 2\theta = \frac{\sqrt{15}}{8}$

d.  $\cos \frac{\theta}{2} = -\sqrt{\frac{3}{8}}$

3. a.  $\theta = \frac{\pi}{2}, \frac{\pi}{2} + \frac{2\pi}{3}, \frac{\pi}{2} + \frac{4\pi}{3}$       b.  $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

4.  $a \approx 2.192$ ,  $B \approx 13.185^\circ$ , and  $C \approx 136.819^\circ$

5. a.  $r = \sin \theta$       b.  $x^2 + y^2 = x + y$

6. a. A circle, the graph is at the end.      b. A line ( $y = 2$ ), the graph is at the end.

7. Graph is at the end.

Domain of  $f = \mathbf{R} - \{\text{all odd integers}\}$       Range of  $f = \mathbf{R}$ ,  
 $x$ -intercepts,  $x = \text{all even integers}$        $y = 0$  is the  $y$ -intercept,  
Period is 2      Vertical asymptotes:  $x = \text{all odd integers}$

8. Graph is at the end.

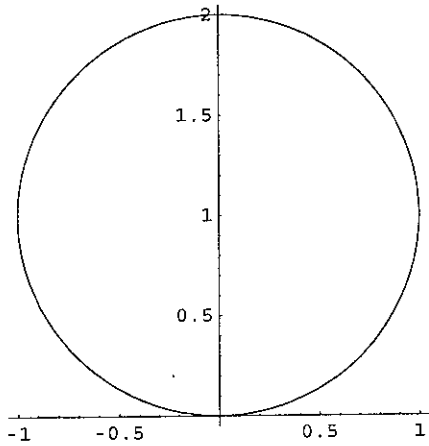
Domain of  $f = \mathbf{R}$ ,      Range of  $f = [-2, 2]$ ,  
Period is  $\frac{2\pi}{3}$ ,      Amplitude is 2,  
Phase shift is  $x = -\frac{\pi}{6}$ .

9. a.  $\sin^{-1}(-1) = \frac{-\pi}{2}$       b.  $\tan^{-1}(1) = \frac{\pi}{4}$       c.  $\sin^{-1}\left(\sin \frac{9\pi}{8}\right) = \frac{-\pi}{8}$

10.  $\tan\left[\sin^{-1}\left(-\frac{1}{3}\right)\right] = -\frac{1}{2\sqrt{2}}$ .

#6 a

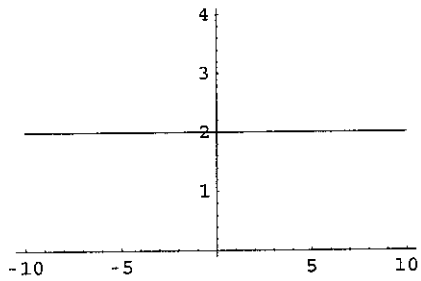
```
r[t_] := 2 * Sin[t];  
ParametricPlot[{r[t] * Cos[t], r[t] * Sin[t]}, {t, 0, 2 * Pi}, AspectRatio -> 1]
```



- Graphics -

#6 b

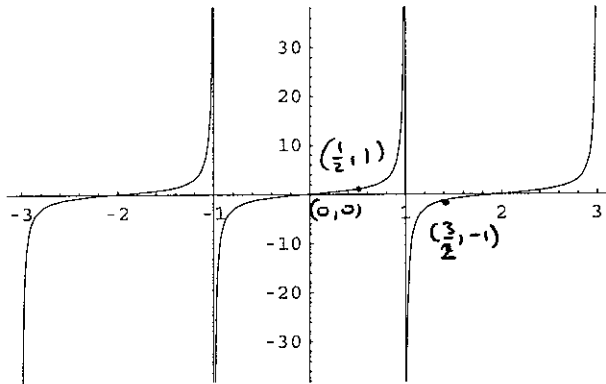
```
ParametricPlot[{t, 2}, {t, -10, 10}]
```



- Graphics -

```
In[11]:= Plot[Tan[x*Pi/2], {x, -3, 3}];
```

# 7



```
In[13]:= Plot[2 Cos[3 x + (Pi/2)], {x, -Pi, Pi}, PlotRange -> {-3, 3}];
```

# 8

