

22m9.Evening section
NEW Practice test for Midterm 2

1. Solve the inequality:

$$\frac{(x-2)^2}{x^2-9} \geq 0$$

2. Let $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x}$

a. Calculate $f \circ g$

b. Find the domains of f, g and $f \circ g$ (identify which is which)

3. Let $f(x) = \frac{2x}{3x-1}$. It is given that f is one-to-one.

a. Calculate the inverse f^{-1} of f

b. Find the domain of f .

c. Find the range of f using f^{-1} .

4. Plot the graph of $f(x) = 5 - e^{-x}$. State its domain, range, intercepts and the horizontal asymptote. Provide 3 points of the graph with both coordinates.

5. Let $f(x) = \ln(x+4)$.

a. Find the domain of f .

b. Graph f .

c. Find all intercepts of f .

d. Determine the range of f and the asymptotes of f .

e. Calculate the inverse f^{-1} of f .

6. a. Find the exact value of the following and give justification for full credit:

$$5^{\log_5 6 + \log_5 7}$$

b. Write the following as a single logarithm and simplify:

$$\ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln(x^2-1)$$

7. Solve the following equations:

a. $2^x = 7$

b. $3 \log_2(x-1) + \log_2 4 = 5$

c. $3^{2x} - 3^{x+1} - 4 = 0$.

8. Let $\alpha = 240^\circ$. Calculate the following.

- Convert α to radians.
- The length of the arc of a circle of radius 3 meters subtended by a central angle α .
- The area of the sector of a circle of radius 3 meters formed by a central angle α .

9. The point $(2, -2)$ is on the terminal side of an angle θ in standard position. Find the exact values of the six trigonometric functions of θ .

10. Find the exact values of the six trigonometric functions of θ , if

$$\tan \theta = \frac{3}{4} \text{ and } \sin \theta < 0.$$

11. Plot the graph of $f(x) = (2 \sin x) + 3$. State its domain, range, period, amplitude and all intercepts. Provide 3 points of the graph with both coordinates.

12. Plot the graph of $f(x) = \tan\left(\frac{\pi}{2}x\right)$. State its domain, range, period, all vertical asymptotes and all intercepts. Provide 3 points of the graph with both coordinates.

13. Plot the graph of $f(x) = 2 \cos\left(3x + \frac{\pi}{2}\right)$. State its domain, range, period, amplitude and phase shift. Provide 3 points of the graph with both coordinates.

14. For the polynomial function $f(x) = -4x^2(x + 2)$:

- Find the x -intercepts, y -intercepts,
- Determine whether the graph of f crosses or touches the x -axis at each x -intercept,
- Determine the behavior of the graph near each x -intercept,
- Describe the end behavior: find the power function that the graph of f resembles for large values of $|x|$,
- Determine the maximum number of turning points,
- Plot the graph of f .

15. Plot the graph of the rational function $f(x) = \frac{-2}{x+1}$ by using transformations (shift, stretch, flip, etc.).

16. For the rational function $f(x) = \frac{3}{x^2-4}$:

- Find the domain,
- Find all vertical asymptotes,
- Find all horizontal asymptotes,
- Find all intercepts,
- Do the sign analysis for f , that is find where the graph of lies above x -axis, and where it lies below the x -axis,
- Plot the graph of $f(x)$.

ANSWERS:

1. $x > 3$ or $x < -3$ or $x = 2$.

2a. $f \circ g = \frac{3}{\frac{2}{x}-1} = \frac{3x}{2-x}$

b. Domain of $f = \{x \in \mathbf{R} : x \neq 1\}$, Domain of $g = \{x \in \mathbf{R} : x \neq 0\}$, and Domain of $f \circ g = \{x \in \mathbf{R} : x \neq 0 \text{ and } x \neq 2\}$

3a. $f^{-1} = \frac{x}{3x-2}$

b. Domain of $f = \{x \in \mathbf{R} : x \neq \frac{1}{3}\}$

c. Range of $f =$ Domain of $f^{-1} = \{x \in \mathbf{R} : x \neq \frac{2}{3}\}$

4. Graph is at the end.

Domain of $f = \mathbf{R}$,

Range of $f = (-\infty, 5)$,

$x = -\ln 5$ is the x -intercept,

$y = 4$ is the y -intercept,

$y = 5$ is the horizontal asymptote

5. Graph is at the end.

Domain of $f = (-4, \infty)$,

Range of $f = \mathbf{R}$,

$x = -3$ is the x -intercept,

$y = \ln 4$ is the y -intercept,

$x = -4$ is the vertical asymptote

$f^{-1}(x) = e^x - 4$

6. a. $5^{\log_5 6 + \log_5 7} = 5^{\log_5 6 \cdot 7} = 6 \cdot 7 = 42$

b. $\ln\left(\frac{x}{x-1}\right)\left(\frac{x+1}{x}\right)\left(\frac{1}{x^2-1}\right) = \ln\left(\frac{1}{x-1}\right)^2 = -2\ln(x-1)$ as long as $x > 1$.

7. a. $x = \log_2 7 = \frac{\ln 7}{\ln 2}$

b. $x = 3$

c. $x = \log_3 4 = \frac{\ln 4}{\ln 3}$

8. a. $\frac{4\pi}{3}$

b. $s = 4\pi$

c. $A = 6\pi$

9. $\cos \theta = \frac{1}{\sqrt{2}}$, $\sin \theta = \frac{-1}{\sqrt{2}}$, $\tan \theta = \cot \theta = -1$, $\sec \theta = \sqrt{2}$, $\csc \theta = -\sqrt{2}$

$$10. \cos \theta = \frac{-4}{5}, \quad \sin \theta = \frac{-3}{5}, \quad \tan \theta = \frac{3}{4} \quad \cot \theta = \frac{4}{3}, \quad \sec \theta = \frac{-5}{4}, \quad \csc \theta = \frac{-5}{3}$$

11. Graph is at the end.

Domain of $f = \mathbf{R}$,

Range of $f = [1, 5]$,

No x -intercepts,

$y = 3$ is the y -intercept,

Period is 2π

Amplitude is 2

12. Graph is at the end.

Domain of $f = \mathbf{R} - \{\text{all odd integers}\}$

Range of $f = \mathbf{R}$,

x -intercepts, $x = \text{all even integers}$

$y = 0$ is the y -intercept,

Period is 2

Vertical asymptotes: $x = \text{all odd integers}$

13. Graph is at the end.

Domain of $f = \mathbf{R}$,

Range of $f = [-2, 2]$,

Period is $\frac{2\pi}{3}$,

Amplitude is 2,

Phase shift is $x = -\frac{\pi}{6}$.

14. a. y -intercept is $y = 0$, and x -intercepts are $x = 0, -2$

b. The graph of f touches the x -axis at $x = 0$, and the graph of f crosses the x -axis at $x = -2$.

c. For x near 0, $-4(x+2)$ is close to -8 , and $f(x) \sim -8x^2$.

For x near -2 , $-4x^2$ is close to -16 , and $f(x) \sim -16(x+2)$.

d. For $|x|$ large, $f(x) = -4x^2(x+2) = -4x^3 - 8x^2 \sim -4x^3$

e. Since the degree of f is 3, there are at most 2 turning points.

g. The graph is at the end.

15. The graph is at the end.

16.

a. The domain is $\{x : x \neq 2 \text{ and } x \neq -2\} = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

b. The vertical asymptotes are $x = 2$ and $x = -2$.

c. The Horizontal asymptote is $y = 0$, since the degree of numerator is less than the degree of denominator.

d. $f(0) = \frac{-3}{4}$ is the y -intercept. There is no x -intercept.

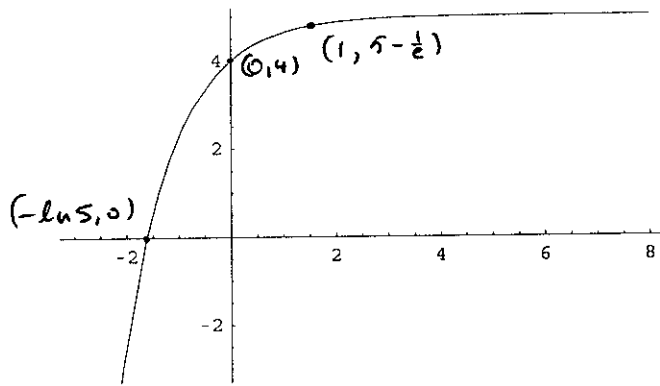
e. The sign analysis

x		-2		2
Interval	$(-\infty, 2)$		$(-2, 2)$	$(2, \infty)$
Test point	-3		0	3
Value at the test point	$f(-3) = \frac{3}{5}$		$f(0) = -\frac{3}{4}$	$f(3) = \frac{3}{5}$
Location of graph	Above x - axis		Below x - axis	Above x - axis

f. The graph is at the end.

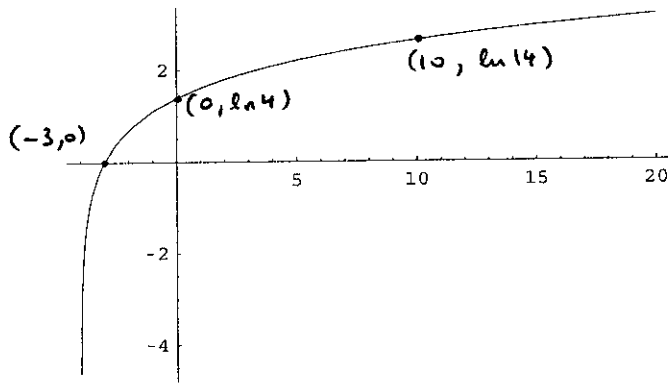
```
In[5]:= Plot[5 - Exp[-x], {x, -3, 8}]
```

#4



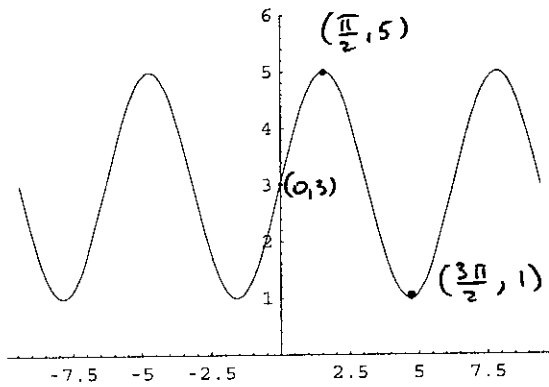
```
In[8]:= Plot[Log[x + 4], {x, -3.99, 20}]
```

#5



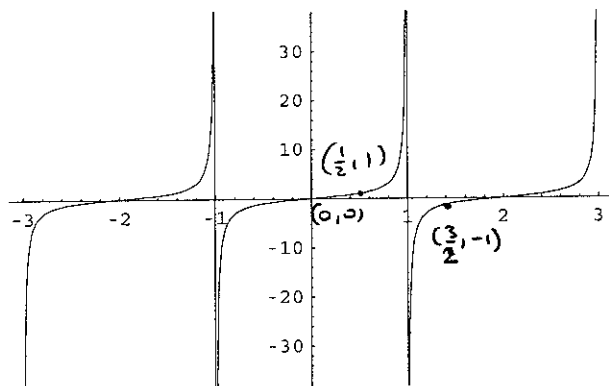
```
In[10]:= Plot[2 Sin[x] + 3, {x, -3 * Pi, 3 * Pi}, PlotRange -> {0, 6}];
```

#11



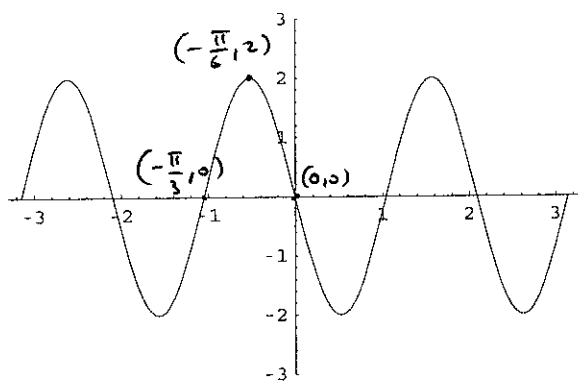
```
In[11]:= Plot[Tan[x*Pi/2], {x, -3, 3}];
```

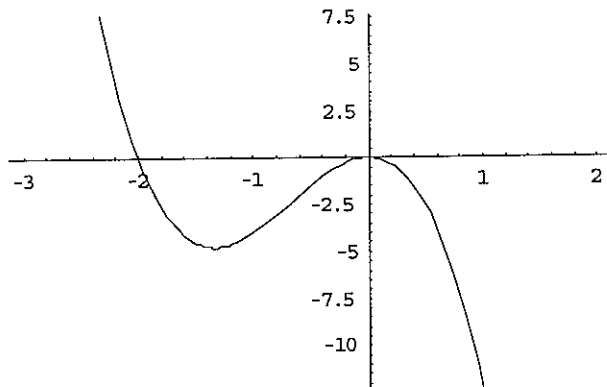
#12



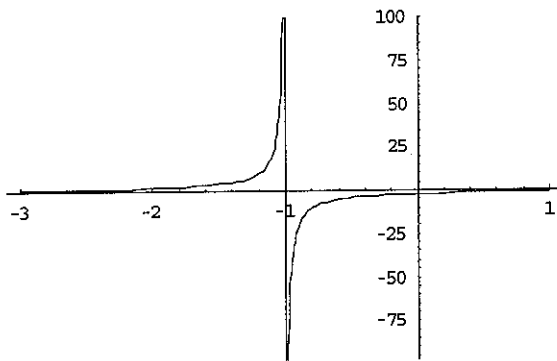
```
In[13]:= Plot[2 Cos[3 x + (Pi/2)], {x, -Pi, Pi}, PlotRange -> {-3, 3}];
```

#13

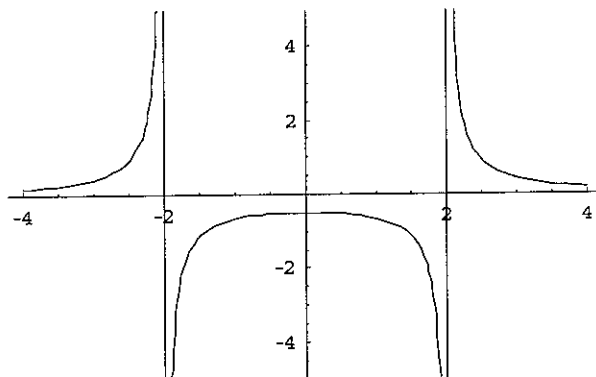




Question 14 above



Question 15 above



Question 16 above