

## 22M:9

### The solution for Practice test for MIDTERM 1

1. Find an equation for the line which is parallel to the line  $2y + x = 3$  and contains the point  $(1, 2)$ .

Answer:  $2y + x = 5$  or  $y = \frac{-1}{2}x + \frac{5}{2}$ .

2. Find the standard form of the equation for the circle with end points of a diameter at  $(1, 4)$  and  $(-3, 2)$ .

Answer:  $(x + 1)^2 + (y - 3)^2 = 5$

3. Let  $f(x) = \sqrt{x}$  and  $g(x) = 3x - 5$ . Find the domains of  $f$ ,  $g$ ,  $f + g$ ,  $f/g$ , and  $g/f$ .

Answers:

Domain of  $f$  is  $\{x : x \geq 0\} = [0, \infty)$

Domain of  $g$  is  $\mathbf{R}$ , all real numbers

Domain of  $f + g$  is  $\{x : x \geq 0\} = [0, \infty)$

Domain of  $\frac{f}{g} = \frac{\sqrt{x}}{3x-5}$  is  $\{x : x \geq 0 \text{ and } x \neq \frac{5}{3}\} = [0, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

Domain of  $\frac{g}{f} = \frac{3x-5}{\sqrt{x}}$  is  $\{x : x \geq 0 \text{ and } x \neq 0\} = (0, \infty)$

4. Plot the graph of the function  $f(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$ . List its domain, range and all  $x$ -intercept(s) and  $y$ -intercept(s). Identify which is which, and state none if there is not one.

Answers:

The graph is in an attached file.

Domain of  $f$  is  $\mathbf{R}$ , all real numbers

Range of  $f$  is  $\mathbf{R}$ , all real numbers

$(0,0)$  is the only  $x$  and  $y$ - intercept.

5. Suppose that a company has just purchased a new computer for \$3000. The company chooses to depreciate the computer using the straight line method over 5 years.

a. Write a linear function that expresses the book value  $V$  of the computer as a function of its age  $x$  (in years).

b. Graph the linear function.

c. What is the book value of the computer after 2 years? After 6 years?

d. When will the computer have book value \$1800?

Answers:

a.  $V(t) = 3000 - 600t$

b. The graph is in an attached file.

c. \$1800 after 2 years, and \$0 after 6 years

d. 2 years

6. The price  $p$  (in dollars) and the quantity  $x$  sold of a certain product obey the demand equation

$$x = -5p + 100, \quad 0 \leq p \leq 20$$

- Express the revenue  $R$  as a function of  $x$ .
- What is the revenue if 15 units are sold?
- What quantity maximizes revenue? What is the maximum revenue?
- What price should the company charge to maximize revenue?

Answers: First solve  $p$  in terms of  $x$  :

$$\begin{aligned} x &= -5p + 100 \\ x - 100 &= -5p \\ 20 - \frac{x}{5} &= p \end{aligned}$$

- Revenue  $= R(x) = xp = x(20 - \frac{x}{5}) = 20x - \frac{x^2}{5}$
- $R(15) = 15 \cdot 17 = \$255$
- $x = -\frac{b}{2a} = \frac{-20}{-2/5} = 50$  units maximizes  $R$ . Maximal value of the revenue is  $R(50) = \$500$ .
- $x = 50$  gives  $p = 20 - \frac{50}{5} = \$10$ , is the optimal price.

7. Let  $f(x) = -x^2 + 3$  and  $g(x) = -3x + 3$ .

- Solve  $f(x) > 0$ .
- Solve  $f(x) > g(x)$ .

Answers:

- $-x^2 + 3 > 0$  has the solution  $-\sqrt{3} < x < \sqrt{3}$
- $-x^2 + 3 > -3x + 3$  gives  $0 > x^2 - 3x$  which has the solution  $0 < x < 3$ .

8. For the polynomial function  $f(x) = -4x^2(x + 2)$  :

- Find the  $x$ -intercepts,  $y$ -intercepts,
- Determine whether the graph of  $f$  crosses or touches the  $x$ -axis at each  $x$ -intercept,
- Determine the behavior of the graph near each  $x$ -intercept,
- Describe the end behavior: find the power function that the graph of  $f$  resembles for large values of  $|x|$ ,
- Determine the maximum number of turning points,
- Plot the graph of  $f$ .

Answers:

- $y$ -intercept is  $y = 0$ , and  $x$ -intercepts are  $x = 0, -2$
- The graph of  $f$  touches the  $x$ -axis at  $x = 0$ , and the graph of  $f$  crosses the  $x$ -axis at  $x = -2$ .
- For  $x$  near 0,  $-4(x + 2)$  is close to  $-8$ , and  $f(x) \sim -8x^2$ .  
For  $x$  near  $-2$ ,  $-4x^2$  is close to  $-16$ , and  $f(x) \sim -16(x + 2)$ .
- For  $|x|$  large,  $f(x) = -4x^2(x + 2) = -4x^3 - 8x^2 \sim -4x^3$
- Since the degree of  $f$  is 3, there are at most 2 turning points.
- The graph is in an attached file.

9. Plot the graph of the rational function  $f(x) = \frac{-2}{x+1}$  by using transformations (shift, stretch, flip, etc.).

The graphs are in an attached file.

10. For the rational function  $f(x) = \frac{3}{x^2-4}$ :

- Find the domain,
- Find all vertical asymptotes,
- Find all horizontal asymptotes,
- Find all intercepts,
- Do the sign analysis for  $f$ , that is find where the graph of lies above  $x$ -axis, and where it lies below the  $x$ -axis,
- Plot the graph of  $f(x)$ .

Answers

- The domain is  $\{x : x \neq 2 \text{ and } x \neq -2\} = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
- The vertical asymptotes are  $x = 2$  and  $x = -2$ .
- The Horizontal asymptote is  $y = 0$ , since the degree of numerator is less than the degree of denominator.

d.  $f(0) = \frac{-3}{4}$  is the  $y$ -intercept. There is no  $x$ - intercept.

e. The sign analysis

$x$	$-2$	$2$	
Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test point	$-3$	$0$	$3$
Value at the test point	$f(-3) = \frac{3}{5}$	$f(0) = -\frac{3}{4}$	$f(3) = \frac{3}{5}$
Location of graph	Above $x$ - axis	Below $x$ - axis	Above $x$ - axis

f. The graph is in an attached file.