

Department of Mathematics

University of Iowa

Iowa City, IA 52242

105 MLH

7:30 am--9:30 am, May 12, 2005

**22M:072**

ELEMENTARY NUMERICAL ANALYSIS

FINAL EXAM

Name: \_\_\_\_\_

Score: \_\_\_\_\_

1. (5%) How to avoid the loss of significance error in computing  $f(x) = (\sqrt{1+x} - 1)/\sin x$  for  $x$  close to 0?

2. (15%) It is known that the equation

$$x - \arctan x = 1$$

has a root  $\alpha \neq 0$ . Propose a convergent method to find  $\alpha$ . Explain why the method is convergent. *Do not attempt to compute the iterates—no credit will be given for that.*

3. (15%) Determine the weights and nodes in the integration formula

$$\int_0^1 f(x) dx \approx w_1 f(0) + w_2 f(x_2)$$

so that the formula is exact for as many polynomials as possible.

4. (10%) Carry out one step of Gaussian elimination with partial pivoting in solving the linear system

$$\begin{aligned}x_1 + x_2 + x_3 &= 5, \\3x_1 + 2x_2 + 2x_3 &= 6, \\4x_1 + 2x_2 + x_3 &= 0.\end{aligned}$$

5. (15%) Compute the LU factorization of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & 2 \\ 0 & 6 & 9 \end{pmatrix}.$$

6. (20%) Consider the system

$$\begin{pmatrix} 4 & -2 & 0 \\ -1 & 4 & -1 \\ 0 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

- (1) Define the Jacobi and GS methods for solving the system.
- (2) Can you determine if the Jacobi and GS methods converge for the given system? Explain.

7. (10%) Consider the initial value problem

$$\begin{aligned}\sin(x + Y(x)) + (1 + x^2) Y'(x) &= 0, \quad 1 \leq x \leq 2, \\ Y(1) &= 10.\end{aligned}$$

Describe, as precisely as possible, the Euler method for solving the problem.

8. (10%) What is A-stability? Give an A-stable method and verify the method is A-stable.