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Title: *Simple curves on surfaces and an analog of a theorem of Magnus*
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Abstract: (joint work with O. Bogopolski and H. Zieschang)

In 1930 Magnus proved that *if elements u, v of a free group $\mathcal{F}$ have the same normal closures, then u is conjugate to $v^{\pm 1}$.* We know the following two generalizations of this theorem.

1) In 1961 Greendlinger proved that if two subsets $\mathcal{U}$ and $\mathcal{V}$ of a free group $\mathcal{F}$ satisfy some small cancellation conditions and have the same normal closures then there is a bijection $\psi\colon \mathcal{U} \to \mathcal{V}$ such that u is conjugate to $\psi(u)^{\pm 1}$.

2) A group is said to be locally indicable if each of its non-trivial, finitely generated subgroups admits an epimorphism onto the infinite cyclic group. Let $\mathcal{A}$ and $\mathcal{B}$ be two non-trivial locally indicable groups. In 1989 Edjvet proved that if $u, v \in \mathcal{A} * \mathcal{B}$ are cyclically reduced words each of length at least two, and if the normal closures of u and v coincide, then u is a conjugate of $v^{\pm 1}$.

Theorem 1. *Let S be a closed surface and g, h non-trivial elements of $\pi_1(S)$ both containing simple closed two-sided curves γ and χ, resp. If h belongs to the normal closure of g then h is conjugate to g^ε or to $(gug^\eta u^{-1})^\varepsilon$, $\varepsilon, \eta \in \{1, -1\}$; here u is a homotopy class containing a simple closed curve μ which properly intersects γ exactly once.*

Moreover, if h is not conjugate to g^ε then $\eta = 1$ if μ is one-sided and $\eta = -1$ elsewise, and χ is homotopic to the boundary of a regular neighbourhood of $\gamma \cup \mu$.

A direct consequence is the following analog of the Magnus' theorem.

Corollary. *Let S be a closed surface and g, h be non-trivial elements of $\pi_1(S)$ both containing simple closed two-sided curves. If the normal closures of g and h coincide then h is conjugate to g or g^{-1}.*

The proof of Theorem 1 is geometrical and uses coverings, intersection numbers of curves, and Brouwer's fixed-point theorem. As a corollary we obtain the following Theorem 2 concerning *normal* automorphisms (an automorphism of a group $\mathcal{G}$ is called normal if it maps each normal subgroup of $\mathcal{G}$ into itself).

Theorem 2. [?] *If S is a closed surface different from the torus and the Klein bottle, then every normal automorphism of $\pi_1(S)$ is an inner automorphism.*

Earlier Lubotsky (1980) and Lue (1980) proved that every normal automorphism of a free group of rank at least 2 is an inner automorphism. In 1996 Neshchadim proved that any normal automorphism of the free product of two non-trivial groups is an inner automorphism.

Remark that Theorem 1 and Corollary admit some analogs for non-simple curves satisfying additional assumptions.