Speaker: **Shin, Satoh**
Title: *Triple point numbers of surface-knots*
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Abstract: The diagrammatic theory of knotted surfaces in 4-space started with Yajima's work in the 1960's. He showed that if a knotted sphere has a projection into 3-space without triple points then it is a ribbon sphere. In the 1980's Giller developed the diagrammatic theory further and Roseman gave seven fundamental moves those are analogous to the Reidemeister moves. More recently, Kamada developed the braid form of a knotted surface following Viro and Rudolph. A series of works due to Carter and Saito in the 1990's are among the most important foundations of the theory.

The triple point number of a knotted surface is the minimal number of triple points among all projections of the surface. This notion is an analogue of the crossing number of a classical knot. There were no examples of non-ribbon spheres whose triple point numbers were concretely determined. We prove that Zeeman's 2- and 3-twist-spun trefoils have the triple point numbers 4 and 6 respectively. Furthermore, infinitely many examples of knotted spheres with triple point number 4 and 6 can be constructed. In the proof we use the quandle cocycle invariants defined by Carter, Jelsovsky, Kamada, Langford, and Saito in 1999.