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Title: *Hilbert's fifth problem and proper actions of Lie groups*

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Abstract: Suppose G is a locally euclidean group and M is a locally euclidean space, and let

$$\Phi : G \times M \longrightarrow M \tag{1}$$

be a continuous action of G on M . In his fifth problem Hilbert asks if one then can choose the local coordinates in G and M so that Φ is real analytic.

When $G = M$ and

$$\Phi : G \times G \longrightarrow G \tag{2}$$

is the multiplication in the group G the answer to Hilbert's question is affirmative, as was proved by Gleason, Montgomery and Zippin.

For the question (1) we prove.

Theorem. Let G be a Lie group which acts on a C^1 smooth manifold M by a C^1 smooth proper action. Then there exists a real analytic structure β on M , compatible with the given smooth structure on M , such that the action of G on M_β is real analytic.

Concerning the uniqueness of β in Theorem 1 we have (from a paper by the author and Marja Kankaanrinta).

Theorem. Let M and N be real analytic proper G -manifolds, where G is a linear Lie group. Suppose that M and N are G -equivariantly C^1 diffeomorphic. Then M and N are G -equivariantly real analytically isomorphic.