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Title: Remarks on Ozsváth-Szabó's Theory
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Abstract: Let $M^{3}=H \bigcup_{F} H$ be a Heegaard diagram of an oriented 3-dimensional homology 3sphere $M^{3}$, where $H$ 's are handlebodies (of genus $g$ ) and $F=\partial H$ is the common boundary surface. The meridians of both sides become the $\alpha$-curves $\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{g}\right)$ and the $\beta$-curves $\left(\beta_{1}, \beta_{2}, \ldots, \beta_{g}\right)$ on $F$. Assume that they intersect transversely. Fix a complex structure of $F$ and it induces a symplectic structure on the $g$-fold symmetric product $\operatorname{Sym}^{g}(F)$. The tori $T_{\alpha}=\alpha_{1} \times \alpha_{2} \times \cdots \times \alpha_{g}$ and $T_{\beta}=\beta_{1} \times \beta_{2} \times \cdots \times \beta_{g}$ are Lagrangians of $\operatorname{Sym}^{g}(T)$. Peter Ozsváth and Zoltán Szaobó have considered the Lagrangian intersection homology theory á la Floer. It turns out that the chain complexes may depend on various choices but the homology groups are the same up to isomorphisms. These are the O-Z invariants of $M^{3}$. For the standard $S^{3}$, it is isomorphic to Z. (We say that O-Z $=1$.)

In a very preliminary joint work with T.J. Li, we try to describe O-Z directly from the $\alpha$-curves and $\beta$-curves on $F$. Particularly, we are experimenting the the following question:

Question: How do we describe the $\alpha$-curves and $\beta$-curves when $\mathrm{O}-\mathrm{Z}=1$ (and $g=2$ )?
We also speculate the relative O-Z invariants. We hope that the relative invariants may be used to study the general case (i.e. $g>2$ ) for $\mathrm{O}-\mathrm{Z}=1$.

