Defn: Let $\{U_1, ..., U_n\}$ be a finite indexed open cover of X. An indexed family of continuous functions

$$\phi_i : X \to [0,1]$$

is a *partition of unity dominated by* $\{U_1, ..., U_n\}$ if
1) support $\phi_i \subset U_i$ for all i.
2) $\Sigma_{i=1}^n \phi_i(x) = 1$ for all x.

Ex: $f_i : \mathbf{R} \to [0,1], f_i(x) = \frac{1}{2}$ is a partition of unity dominated by $U_i = \mathbf{R}$, $i = 1, 2$

Ex: $\phi_i : \mathbf{R} \to [0,1]$,

$$\phi_1(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}, \qquad \phi_2(x) = \begin{cases} 1 & \text{if } x < 0 \\ 1-x & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}.$$

is a partition of unity dominated by

$$U_1 = (-1, \infty), \qquad U_2 = (\infty, 2)$$

Note: partition of unity for an arbitrary open cover will be defined in section 41 (one more condition, which finite covers automatically satisfy, will be needed).

Thm 36.1: (Existence of finite partitions of unity): Suppose X is T_4 and $X \subset \cup_{i=1}^n U_i^{open}$. Then there exists a partition of unity dominated by $\{U_1, ..., U_n\}$

Thm 36.2: X compact m-mfld, then X can be imbedded in $\mathbf{R}^N$ for some $\mathbf{N} \in \mathbf{Z}$.

Section 39:

A collection $\mathcal{A}$ of subsets of X is *locally finite* if for all $x \in X$, there exists U open such that $x \in U$ and U intersects only finitely many elements of $\mathcal{A}$

Ex: $\mathcal{A} = \{(n, n+2) \mid n \in \mathbf{Z}\}$ is locally finite.

Ex: $\mathcal{C} = \{(n, n+2) \mid n \in \mathbf{Z}_+\}$ is locally finite.

Ex: $\mathcal{D} = \{(0, n) \mid n \in \mathbf{Z}_+\}$ is NOT locally finite.

Ex: A finite collection of sets is locally finite.

The indexed family $\{A_\alpha \mid \alpha \in J\}$ is a *locally finite indexed family* in X if for all $x \in X$, there exists U open such that $x \in U$ and U intersects A_α for only finitely many α.

Ex: If $A_i = \mathbf{R}$ for all $i \in \mathbf{Z}$, then $\{A_i \mid i \in \mathbf{Z}\}$ is NOT a locally finite indexed family in X, but $\{A_i \mid i \in \mathbf{Z}\}$, as a collection of set(s), is locally finite (since it contains only one set).