Homework 1 (first part) [adapted from latex HW of Colin McKinney]

1.0 Show $X - \bigcap_{i=1}^{n} U_i = \bigcup_{i=1}^{n}(X - U_i)$

$x \in X - \bigcap_{i=1}^{n} U_i$

iff $x \in X$ and $x \notin \bigcap_{i=1}^{n} U_i$

iff $x \in X$ and $x \notin U_i$ for some i

iff $x \in X - U_i$ for some i

iff $x \in \bigcup_{i=1}^{n}(X - U_i)$.

1.1 Let X be a toplogical space; let A be a subset of X. Suppose that for each $x \in A$ there is an open set U_x containing x such that $U_x \subset A$. Show that A is open in X.

Proof: For each $x \in A$, take an open set U_x containing x such that $U_x \subset A$ (U_x exists by hypothesis). Hence $\cup_{x\in A} U_x$ is also open (by defn of topology). $\cup_{x\in A} U_x = A$, and so A must also be open.