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**Off-diagonal terms in symmetric operators. (English. English summary)**

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A densely defined unbounded symmetric operator $S$ in a Hilbert space $\mathcal{H}$ is said to be smooth if there is a sequence of projections $P_1 \leq P_2 \leq \cdots$ such that $P_j\mathcal{H} \subset \mathrm{dom}S$ and $\sup_j P_j = I$. One can consider a smooth operator $S$ as a generalization of a Jacobi matrix if it satisfies additional assumptions

$$\text{(A)} \qquad (I - P_{j+1})SP_j = 0 \quad \text{for all } j = 1, 2, \cdots.$$

In the paper sufficient conditions for such an operator $S$ to be essentially selfadjoint are given in terms of the off-diagonal terms $B_j = (I - P_j)SP_j$. One of them is similar to that for the Jacobi matrix: for a smooth operator $S$ satisfying (A), to be essentially selfadjoint it is sufficient that $\sum_{j=1}^{\infty} \|B_j\|^{-1} = \infty$. A more subtle sufficient condition has a similar form with norm estimates on $B_j$ replaced by estimates on a sequence $B_jx,\ x \in \mathcal{H}$. For a positive symmetric operator $L$ a sufficient condition is given in terms of the sequence $d_j(x) = \langle B_jx, x\rangle,\ j \in \mathbf{N}$: if $\sum d_j(x)^{-1} = \infty$ for every $x \in \mathcal{H}$ then a positive smooth operator $L$ satisfying (A) is essentially selfadjoint. This result is applied to a finite family of commuting symmetric operators $\{S_j\}_{j=1}^k$. In view of Nelson's theorem it can be reformulated as a sufficient condition for the operators $S_j$ to be essentially selfadjoint and for the family $\{S_j\}_{j=1}^k$ to have a joint resolution of identity. *Vladimir Derkach* (UKR-DONE)