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Unitary representations of Lie groups with reflection symmetry. (English. English summary)

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Let G be a real Lie group with a nontrivial involutive automorphism τ. A unitary representation π of G on a Hilbert space H is said to be reflection symmetric if there exists an involutive unitary operator J on H such that $\pi(\tau(g)) = J\pi(g)J$, $g \in G$. Denote $H = G^\tau$. The Lie algebra $\mathfrak{g}$ of G admits the decomposition $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$, where $\mathfrak{q}$ is the eigenspace of $d\tau$ corresponding to -1. Assume a closed H-invariant convex cone $C \subset \mathfrak{q}$ to be given such that: $C^\circ \neq \varnothing$; $\mathrm{ad}Y$ is $\mathbf{R}$-semisimple for all $Y \in C^\circ$; $S(C) = H \exp C$ is a closed semigroup, and $H \exp C^\circ$ is diffeomorphic to $H \times C^\circ$; there is a closed $S(C)$-invariant subspace $0 \neq K_0 \subset H$ satisfying $\langle v | J(v) \rangle \geq 0$ for all $v \in K_0$. Let G^c denote the simply connected Lie group with the Lie algebra $\mathfrak{g}^c = \mathfrak{h} \oplus i\mathfrak{q}$. The main result of the paper is a construction of a unitary representation π^c of G^c acting on a quotient of K_0 such that $d\pi^c(X)$ is induced by $d\pi(X)$ for $X \in \mathfrak{h}$ and $id\pi^c(Y)$ is induced by $d\pi(iY)$ for $Y \in C$. This construction applies to the case when G is semisimple and G/H is a non-compactly causal symmetric space and, in particular, a Cayley-type space, the representation π^c being an irreducible unitary highest weight representation. Finally, the semidirect products $G = HN$, where N is normal and abelian, are considered.

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